

CHARMLESS $B \rightarrow VP$ DECAYS USING FLAVOR SU(3) SYMMETRY *

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The decays of B mesons to a charmless vector (V) and pseudoscalar (P) meson are analyzed within a framework of flavor SU(3) in which symmetry breaking is taken into account through ratios of decay constants in tree (T) amplitudes. The magnitudes and relative phases of tree and penguin amplitudes are extracted from data; the symmetry assumption is tested; and predictions are made for rates and CP asymmetries in as-yet-unseen decay modes. A key assumption for which we perform some tests and suggest others is a relation between penguin amplitudes in which the spectator quark is incorporated into either a pseudoscalar meson or a vector meson. Values of γ slightly restricting the range currently allowed by fits to other data are favored, but outside this range there remain acceptable solutions which cannot be excluded solely on the basis of present $B \rightarrow VP$ experiments.

I. INTRODUCTION

B meson decays are a rich source of information on fundamental phases of weak charge-changing couplings, as encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Decays to charmless final states, many of which occur at branching ratios exceeding 10^{-5} , are particularly useful, since many of them involve more than one significant quark subprocess and thus have the potential for displaying direct CP asymmetries. To interpret such data one must disentangle information on CKM (weak) phases from strong-interaction final-state phases.

In $B \rightarrow PP$ decays, where P is a charmless pseudoscalar meson, flavor SU(3) symmetries have been employed [1, 2, 3, 4, 5] to extract weak phases in such decays as $B^0 \rightarrow \pi^+\pi^-$ and various charge states of $B \rightarrow K\pi$ (see, e.g., the recent reviews of [6] and a recent analysis [7] of $B \rightarrow K\pi$). The decays $B \rightarrow VP$, where V is a charmless vector meson, involve more invariant amplitudes, since one cannot use Bose statistics to simplify the decays, in contrast to the case of two spinless final pseudoscalars in the same meson multiplet [1, 2, 3, 8]. Nonetheless, after the first report of a charmless $B \rightarrow VP$ decay [9], it became possible to perform such analyses by using rates and CP asymmetries in some decays to predict others [8, 10, 11, 12].

In the present paper, following upon our recent analysis of $B \rightarrow PP$ decays [13], we analyze $B \rightarrow VP$ decays within flavor SU(3), incorporating symmetry breaking through ratios of meson decay constants in tree (T) am-

plitudes. The magnitudes and relative phases of invariant amplitudes are extracted from data; the symmetry assumption is tested; and predictions are made for rates and CP asymmetries in as-yet-unseen decay modes.

Our approach differs from ones involving *a priori* calculations of $B \rightarrow VP$ decay rates and CP asymmetries involving QCD and factorization. Factorization was applied to these decays in Refs. [14, 15, 16, 17]. The QCD factorization methods of Refs. [18, 19] were considered for $B \rightarrow VP$ decays in Refs. [19, 20, 21, 22, 23, 24]. Many of these authors were able to fit some data but could not reproduce those processes dominated by strangeness-changing penguin amplitudes, which others have argued should be enhanced [25, 26, 27, 28]. Our method, by contrast, relies on assumptions of isospin and SU(3) flavor symmetry, provides tests of these assumptions, and is capable of extracting strong final-state phases from data rather than needing to predict them. It is similar to the analysis of $B \rightarrow \rho^\pm\pi^\mp$ and $B \rightarrow \rho^\mp K^\pm$ in Ref. [29] and of $B \rightarrow K^{*\pm}\pi^\mp$ in Ref. [30] (which uses extensive data on $B \rightarrow \rho^\pm\pi^\mp$), but we are concerned with a wider set of $B \rightarrow VP$ decays.

The present analysis has considerable sensitivity to the CKM phase γ . This is driven in part by the pattern of tree-penguin interference in a wide variety of $B \rightarrow VP$ decays, and in part by the incorporation of time-dependent information on $B \rightarrow \rho\pi$, as has also been noted in Ref. [24].

We review notation and conventions for amplitudes in Section II. We average currently known experimental rates and CP asymmetries from the CLEO, BaBar, and Belle Collaborations and use these averages to obtain magnitudes of amplitudes in Section III. We then show how to extract invariant amplitudes (identified with specific flavor topologies) in Section IV by fitting the experimental amplitudes and CP asymmetries. The simplest fit assumes a relation between penguin amplitudes [31] in which the spectator quark is incorporated into either a pseudoscalar meson or a vector meson. We sug-

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gest specific tests of this assumption in Section V. It is relaxed in Section VI to see if the quality of the overall fit improves, and predictions for rates and CP asymmetries for observed and as-yet-unseen $B \rightarrow VP$ modes are discussed. Relations among amplitudes based on the U-spin subgroup of $SU(3)$ are presented in Section VII, while Section VIII concludes. An Appendix discusses $B \rightarrow \rho^\mp \pi^\pm$ rates and asymmetries.

II. NOTATION

We use the following quark content and phase conventions:

- *Bottom mesons:* $B^0 = d\bar{b}$, $\bar{B}^0 = b\bar{d}$, $B^+ = u\bar{b}$, $B^- = -b\bar{u}$, $B_s = s\bar{b}$, $\bar{B}_s = b\bar{s}$;
- *Charmed mesons:* $D^0 = -c\bar{u}$, $\bar{D}^0 = u\bar{c}$, $D^+ = c\bar{d}$, $D^- = d\bar{c}$, $D_s^+ = c\bar{s}$, $D_s^- = s\bar{c}$;
- *Pseudoscalar mesons:* $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$;
- *Vector mesons:* $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $\bar{K}^{*0} = s\bar{d}$, $K^{*-} = -s\bar{u}$, $\phi = s\bar{s}$.

In the present approximation there are four types of independent amplitudes: a “tree” contribution t ; a “color-suppressed” contribution c ; a “penguin” contribution p ; and a “singlet penguin” contribution s , in which a color-singlet $q\bar{q}$ pair produced by two or more gluons or by a Z or γ forms an $SU(3)$ singlet state. We neglect smaller contributions from an “exchange” amplitude e , an “annihilation” amplitude a , and a “penguin annihilation” amplitude pa . The amplitudes we retain contain both the leading-order and electroweak penguin contributions:

$$\begin{aligned} t &\equiv T + P_{EW}^C, & c &\equiv C + P_{EW}, \\ p &\equiv P - \frac{1}{3}P_{EW}^C, & s &\equiv S - \frac{1}{3}P_{EW}, \end{aligned} \quad (1)$$

where the capital letters denote the leading-order contributions ([4, 5, 32, 33]) while P_{EW} and P_{EW}^C are respectively color-favored and color-suppressed electroweak penguin (EWP) amplitudes [5]. We shall denote $\Delta S = 0$ transitions by unprimed quantities and $|\Delta S| = 1$ transitions by primed quantities. For VP decay modes, the subscript V or P denotes the final-state meson (vector or pseudoscalar) incorporating the spectator quark. Thus, for example, a color-favored $\Delta S = 0$ tree amplitude in which the spectator quark is incorporated into a pseudoscalar meson will be denoted t_P . Although some $B \rightarrow VV$ decay processes have been seen, we shall not discuss them further here.

For the $\bar{b} \rightarrow \bar{d}$ and $\bar{b} \rightarrow \bar{u}u\bar{d}$ transitions, an educated guess of the hierarchies among the amplitudes is given

in Ref. [5, 12, 13]. For $|\Delta S| = 1$ transitions, c' contains an electroweak penguin amplitude at the next order. Therefore, we put c' together with t' at the same order. Similarly, since part of the singlet amplitude is the electroweak penguin, s' is at least of order P'_{EW} .

The partial decay width of two-body B decays is

$$\Gamma(B \rightarrow M_1 M_2) = \frac{p_c}{8\pi m_B^2} |\mathcal{A}(B \rightarrow M_1 M_2)|^2, \quad (2)$$

where p_c is the momentum of the final state meson in the rest frame of B , m_B is the B meson mass, and M_1 and M_2 can be either pseudoscalar or vector mesons. Using Eq. (2), one can extract the magnitude of the invariant amplitude of each decay mode from its experimentally measured branching ratio. To relate partial widths to branching ratios, we use the world-average lifetimes $\tau^+ = (1.653 \pm 0.014)$ ps and $\tau^0 = (1.534 \pm 0.013)$ ps computed by the LEPBOSC group [34]. Unless otherwise indicated, for each branching ratio quoted we imply the average of a process and its CP -conjugate.

Two phase conventions are in current use for the penguin amplitudes, depending on whether one considers them to be dominated by the CKM factors $V_{tb}^* V_{tq}$ ($q = s, d$), or integrates out the t quark, uses the unitarity relation $V_{tb}^* V_{tq} = -V_{cb}^* V_{cq} - V_{ub}^* V_{uq}$, and absorbs the $V_{ub}^* V_{uq}$ term into a redefined tree amplitude. Here we adopt the latter convention. For a discussion of the relation between the two see, e.g., Ref. [35]. Thus both the strangeness-changing and strangeness-preserving penguin amplitudes will be taken to have real weak phases in this discussion.

III. EXPERIMENTAL DATA AND AMPLITUDE DECOMPOSITIONS

The experimental branching ratios and CP asymmetries from the CLEO, BaBar, and Belle Collaborations are summarized and averaged in Tables I (for $\Delta S = 0$ transitions) and II (for $|\Delta S| = 1$ transitions). Data are current up to and including the 2003 Lepton-Photon Symposium at Fermilab. We use the Particle Data Group method [36] for performing averages, including a scale factor $S \equiv [\chi^2/(N-1)]^{1/2}$ when the χ^2 for an average of N data points exceeds $N-1$. (The Heavy Flavor Averaging Group [37] does not use this scale factor. In other respects our averages agree with theirs when inputs are the same.) The corresponding experimental amplitudes, extracted from partial decay rates using Eq. (2), are shown in Tables III and IV. In these tables we also give the theoretical expressions for these amplitudes (see also Refs. [8, 10, 12]) and, anticipating the results of the next section, the magnitudes of contributions to the observed amplitude of the invariant amplitudes $|T|$ and $|\mathcal{P}|$ or $|T'|$ and $|\mathcal{P}'|$ in one fit to the data. These contributions include Clebsch-Gordan coefficients. CP

asymmetries are defined as

$$A_{CP}(B \rightarrow f) \equiv \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)}. \quad (3)$$

By comparing the magnitudes of individual contributions with experimental amplitudes, one can tell whether one contribution dominates or whether constructive or destructive interference between two contributions is favored.

IV. EXTRACTING AMPLITUDES

In the present section we show how a global fit to decay rates and CP asymmetries can determine many (though not all) of the invariant amplitudes governing $B \rightarrow VP$ decays. We shall be able to determine from experimental data their magnitudes and relative strong phases and the weak phase γ . We shall assume a universal ratio $p'_V/p'_P = -ce^{i\phi}$, initially assuming $c = 1$ and $\phi = 0$ in accord with Ref. [31], presenting also results with arbitrary c and ϕ . We interpret ϕ as a relative *strong* phase between p'_V and $-p'_P$, so that it does not change sign under CP -conjugation. We now explain in some detail the inputs and fit parameters.

We base the present fit on the following processes (see Tables III and IV):

- The $B^+ \rightarrow K^{*0}\pi^+$ amplitude involves $|p'_P|$ alone. The decay rate provides one data point. No CP asymmetry is expected or seen.
- The decays $B^0 \rightarrow \rho^-\pi^+$ and $\bar{B}^0 \rightarrow \rho^+\pi^-$ (equivalently, their CP -averaged branching ratio and CP asymmetry quoted in Table I) involve t_V and p_V . These processes thus provide two data points.
- The decays $B^0 \rightarrow \rho^+\pi^-$ and $\bar{B}^0 \rightarrow \rho^-\pi^+$ involve t_P and p_P and provide two data points.
- In the time-dependent study of $(B^0, \bar{B}^0) \rightarrow \rho^\mp\pi^\pm$, the asymmetry parameters S_{+-} and S_{-+} , to be defined at the end of this section, provide two more data points. (Other time-dependent parameters are related to those already included.)
- The decays $B^0 \rightarrow \rho^-K^+$ and $\bar{B}^0 \rightarrow \rho^+K^-$ involve t'_V and p'_V and provide two data points, since the CP -averaged decay rate and CP asymmetry have been presented.
- The decays $B^0 \rightarrow K^{*+}\pi^-$ and $\bar{B}^0 \rightarrow K^{*-}\pi^+$ involve t'_P and p'_P and similarly provide two data points.
- The decays $B \rightarrow K^*\eta$ (for both charge states) play an important role in constraining the phase ϕ of $-p'_V/p'_P$, since this phase must be small in

order that p'_P and p'_V contribute constructively to the large decay rate, as anticipated in Ref. [31]. We include two decay rates and two CP asymmetries, adding a total of four data points. Since our scheme predicts a very small CP asymmetry for $B^0 \rightarrow K^{*0}\eta$, the parameters of the fit will not be affected by this observable.

- The rate and CP asymmetry for $B^+ \rightarrow \rho^0\pi^+$ and $B^+ \rightarrow \omega\pi^+$ have been measured. The two decay rates are dominated by t_V but provide some information about the magnitude of the amplitude c_P , about which we shall have more to say below. These processes thus add four more data points to our fits.
- The rate and CP asymmetry for $B^+ \rightarrow \rho^+\pi^0$ have been measured, adding two data points.
- The decay rates for $B \rightarrow \phi K$ (both charge states) have been measured. The corresponding decay widths are expected to be equal. They are measured to be within 7% when one takes into account the difference in lifetimes of the B^+ and B^0 mesons. [Note added: The branching ratio $\mathcal{B}(B^0 \rightarrow \phi K^0)$ quoted in Table II has been updated [47]. The central values of the world-averaged decay widths now are exactly equal.] We include the $B \rightarrow \phi K$ decay rates as two more data points. Since both the amplitudes p'_P and s'_P contributing to these processes are expected to have the same weak phase, we predict zero CP asymmetry in any $B \rightarrow \phi K$ decay. This is certainly true for the charged mode, whose CP asymmetry we include as another data point.
- Taking the average of BaBar and Belle values [64], we find the time-dependent parameters in $B^0 \rightarrow \phi K_S$ to be $S_{\phi K_S} = -0.147 \pm 0.697$ ($S = 2.11$) and $A_{\phi K_S} = 0.046 \pm 0.256$ ($S = 1.08$), whereas we predict the standard model values $(\sin 2\beta, 0)$. The average of BaBar and Belle determinations via the subprocess $\bar{b} \rightarrow \bar{c}c\bar{s}$ is $\sin 2\beta = 0.736 \pm 0.049$ [64]. The parameter $A_{\phi K_S}$ is equivalent to the direct CP asymmetry $A_{CP}(B^0 \rightarrow \phi K_S)$. (The corresponding asymmetry for $B^+ \rightarrow \phi K^+$ is seen in Table II to be very small.) These observables thus contribute $\Delta\chi^2 = (1.61, 0.03)$ to our fit, without affecting the fit parameters. We include them in our χ^2 total, adding two more data points. In view of the large S -factor, contributing to a considerable amplification of the experimental error in $S_{\phi K_S}$, it is premature to regard the deviation of this observable from its standard model expectation as a signal of new physics. In Ref. [65] we discussed some scenarios which could give rise to such a deviation. One should add a penguin amplitude (e.g., for $\bar{b} \rightarrow \bar{s}s\bar{s}$) with arbitrary magnitude and weak and strong phases to the present global fit to see if one can describe all the $B \rightarrow VP$ data with any greater success.

TABLE I: Experimental branching ratios of selected $\Delta S = 0$ decays of B mesons. CP -averaged branching ratios are quoted in units of 10^{-6} . Numbers in parentheses are upper bounds at 90 % c.l. References are given in square brackets. Additional line, if any, gives the CP asymmetry A_{CP} . The error in the average includes the scale factor S when this number is shown in parentheses.

	Mode	CLEO	BaBar	Belle	Avg.
$B^+ \rightarrow$	$\bar{K}^{*0} K^+$	$0.0^{+1.3+0.6}_{-0.0-0.0} (< 5.3)$ [38]	-	-	< 5.3
	$\rho^0 \pi^+$	$10.4^{+3.3}_{-3.4} \pm 2.1$ [38]	$9.3 \pm 1.0 \pm 0.8$ [39]	$8.0^{+2.3}_{-2.0} \pm 0.7$ [40]	9.1 ± 1.1
		-	$-0.17 \pm 0.11 \pm 0.02$ [39]	-	-0.17 ± 0.11
	$\rho^+ \pi^0$	< 43 [38]	$11.0 \pm 1.9 \pm 1.9$ [39]	-	11.0 ± 2.7
		-	$0.23 \pm 0.16 \pm 0.06$ [39]	-	0.23 ± 0.17
	$\rho^+ \eta$	$4.8^{+5.2}_{-3.8} (< 15)$ [41]	$10.5^{+3.1}_{-2.8} \pm 1.3$ [42]	< 6.2 [43]	8.9 ± 2.7
		-	$0.06 \pm 0.29 \pm 0.02$ [42]	-	0.06 ± 0.29
	$\rho^+ \eta'$	$11.2^{+11.9}_{-7.0} (< 33)$ [41]	$14.0^{+5.1}_{-4.6} \pm 1.9$ [42]	-	13.3 ± 4.5
	$\omega \pi^+$	$11.3^{+3.3}_{-2.9} \pm 1.4$ [38]	$5.4 \pm 1.0 \pm 0.5$ [44]	$5.7^{+1.4}_{-1.3} \pm 0.6$ [45]	5.9 ± 1.1 ($S = 1.23$)
		$-0.34 \pm 0.25 \pm 0.02$ [46]	$0.04 \pm 0.17 \pm 0.01$ [44]	$0.48^{+0.23}_{-0.20} \pm 0.02$ [45]	0.10 ± 0.21 ($S = 1.84$)
$B^0 \rightarrow$	$\phi \pi^+$	< 5 [9]	< 0.41 [47]	-	< 0.41
	$\rho^\mp \pi^\pm$	$27.6^{+8.4}_{-7.4} \pm 4.2$ [38]	$22.6 \pm 1.8 \pm 2.2$ [48]	$29.1^{+5.0}_{-4.9} \pm 4.0$ [49]	24.0 ± 2.5
		-	$-0.11 \pm 0.06 \pm 0.03$ [50]	$-0.38^{+0.19+0.04}_{-0.21-0.05}$ [49]	-0.14 ± 0.08 ($S = 1.31$)
	$\rho^- \pi^+$	-	9.5 ± 2.0^a	-	10.2 ± 2.0^b
		-	$-0.52^{+0.17}_{-0.19} \pm 0.07$ [50]	-	-0.54 ± 0.19^b
	$\rho^+ \pi^-$	-	13.1 ± 2.3^a	-	13.8 ± 2.2^b
		-	$-0.18 \pm 0.13 \pm 0.05$ [50]	-	-0.16 ± 0.15^b
	$\rho^0 \pi^0$	$1.6^{+2.0}_{-1.4} \pm 0.8$ (< 5.5) [38]	$0.9 \pm 0.7 \pm 0.5$ (< 2.5) [39]	$6.0^{+2.9}_{-2.3} \pm 1.2$ [49]	< 2.5
	$\rho^0 \eta$	$2.6^{+3.2}_{-2.6} (< 10)$ [41]	-	< 5.5 [51]	< 5.5
	$\rho^0 \eta'$	$0.0^{+5.8}_{-0.0} (< 12)$ [41]	-	< 14 [43]	< 12
	$\omega \pi^0$	$0.8^{+1.9+1.0}_{-0.8-0.8} (< 5.5)$ [38]	$-0.3 \pm 1.1 \pm 0.3$ (< 3) [52]	< 1.9 [45]	< 1.9
	$\omega \eta$	< 12 [9]	-	-	< 12
	$\omega \eta'$	< 60 [9]	-	-	< 60
	$\phi \pi^0$	< 5 [9]	-	-	< 5
	$\phi \eta$	< 9 [9]	-	-	< 9
	$\phi \eta'$	< 31 [9]	< 1.0 [53]	-	< 1.0

^aBased on asymmetries quoted in Ref. [50] and BaBar value of $\mathcal{B}(B^0 \rightarrow \rho^\mp \pi^\pm)$.

^bBased on asymmetries quoted in Ref. [50] and world averages for \mathcal{B} and A_{CP} for $B^0 \rightarrow \rho^\mp \pi^\pm$.

- Both a decay rate and a CP asymmetry have been presented for $B^+ \rightarrow \omega K^+$, while we are aware only of a decay rate for $B^0 \rightarrow \omega K^0$. We thus add three more data points for these processes.
- The BaBar Collaboration has recently reported observation of the decay modes $B^+ \rightarrow \rho^+ \eta$ and $B^+ \rightarrow \rho^+ \eta'$ [42] at levels indicating a significant role for the C_V amplitude. We include the branching ratios for these processes as averages between the BaBar and older CLEO [41] values. In addition we include the new BaBar value of $A_{CP}(B^+ \rightarrow \rho^+ \eta)$.
- The decay rate for $B^+ \rightarrow \rho^0 K^+$ was recently measured by the Belle collaboration with high significance [55]. We include the average between the Belle result and the previous measurements by BaBar and CLEO as another data point.
- Although only an upper limit exists so far for

$\mathcal{B}(B^0 \rightarrow K^{*0} \pi^0)$, we use the Belle central value and error [62] in order to enforce this upper limit in the fits.

The grand total of fitted data points is thus 34, including some quantities such as $A_{CP}(B^+ \rightarrow \phi K^+)$, $S_{\phi K_S}$ and $A_{\phi K_S}$ which do not affect our fit. We now count the parameters of the fit.

- The amplitude p'_P is taken to have a strong phase of π by definition. Its weak phase, since it is dominated by $-V_{cb}^* V_{cs}$ (see the discussion at the end of Sec. II), also is π , so we will have p'_P real and positive. This choice will be seen in our favored solution to entail tree amplitudes with positive real and small imaginary parts (when their weak phases are neglected), in accord with expectations from factorization [24]. The corresponding strangeness-preserving amplitude p_P is determined by $p_P = (V_{cd}/V_{cs})p'_P = -\bar{\lambda}p'_P$, where $\bar{\lambda} \equiv$

TABLE II: Same as Table II for $|\Delta S| = 1$ decays of B mesons.

	Mode	CLEO	BaBar	Belle	Avg.
$B^+ \rightarrow$	$K^{*0}\pi^+$	$7.6^{+3.5}_{-3.0} \pm 1.6$ (< 16) [38]	$15.5 \pm 1.8^{+1.5}_{-4.0}$ [54]	$8.5 \pm 0.9^{+0.8+0.8}_{-0.7-0.5}$ [55]	9.0 ± 1.4 ($S = 1.11$)
	$K^{*+}\pi^0$	$7.1^{+11.4}_{-7.1} \pm 1.0$ (< 31) [38]	-	-	< 31
	$K^{*+}\eta$	$26.4^{+9.6}_{-8.2} \pm 3.3$ [41]	$25.7^{+3.8}_{-3.6} \pm 1.8$ [42]	$26.5^{+7.8}_{-7.0} \pm 3.0$ [56]	25.9 ± 3.4
	-	-	$0.15 \pm 0.14 \pm 0.02$ [42]	$-0.05^{+0.25}_{-0.30} \pm 0.01$ [57]	0.10 ± 0.12
	$K^{*+}\eta'$	$11.1^{+12.7}_{-8.0}$ (< 35) [41]	$6.1^{+3.9}_{-3.2} \pm 1.2$ (< 12) [42]	< 90 [43]	< 12
	$\rho^0 K^+$	$8.4^{+4.0}_{-3.4} \pm 1.8$ (< 17) [38]	$3.9 \pm 1.2^{+1.3}_{-3.5}$ (< 6.2) [54]	$3.9 \pm 0.6^{+0.4+0.7}_{-0.3-0.2}$ [55]	4.1 ± 0.8
	$\rho^+ K^0$	< 48 [58]	-	-	< 48
	ωK^+	$3.2^{+2.4}_{-1.9} \pm 0.8$ (< 7.9) [38]	$5.0 \pm 1.0 \pm 0.4$ [44]	$6.7^{+1.3}_{-1.2} \pm 0.6$ [45]	5.4 ± 0.8
	-	-	$-0.05 \pm 0.16 \pm 0.01$ [44]	$0.06^{+0.20}_{-0.18} \pm 0.01$ [45]	0.00 ± 0.12
	ϕK^+	$5.5^{+2.1}_{-1.8} \pm 0.6$ [59]	$10.0^{+0.9}_{-0.8} \pm 0.5$ [47]	$8.6 \pm 0.8^{+0.6+0.0}_{-0.6-0.3}$ [55]	9.0 ± 0.9 ($S = 1.39$)
	-	-	$0.039 \pm 0.086 \pm 0.011$ [47]	$0.01 \pm 0.12 \pm 0.05$ [60]	0.03 ± 0.07
$B^0 \rightarrow$	$K^{*+}\pi^-$	$16^{+6}_{-5} \pm 2$ [61]	-	$14.8^{+4.6+1.5+2.4}_{-4.4-1.0-0.9}$ [62]	15.3 ± 3.8
	-	$0.26^{+0.33+0.10}_{-0.34-0.08}$ [63]	-	-	0.26 ± 0.35
	$K^{*0}\pi^0$	$0.0^{+1.3+0.5}_{-0.0-0.0}$ (< 3.6) [38]	-	$0.42^{+1.85}_{-1.74} \pm 0.06$ (< 3.5) [62]	0.4 ± 1.8^a (< 3.5)
	$K^{*0}\eta$	$13.8^{+5.5}_{-4.6} \pm 1.6$ [41]	$19.0^{+2.2}_{-2.1} \pm 1.3$ [42]	$16.5^{+4.6}_{-4.2} \pm 1.2$ [56]	17.8 ± 2.0
	-	-	$0.03 \pm 0.11 \pm 0.02$ [42]	$0.17^{+0.28}_{-0.25} \pm 0.01$ [57]	0.05 ± 0.10
	$K^{*0}\eta'$	$7.8^{+7.7}_{-5.7}$ (< 24) [41]	$3.2^{+1.8}_{-1.6} \pm 0.9$ (< 6.4) [42]	< 20 [43]	< 6.4
	$\rho^- K^+$	$16.0^{+7.6}_{-6.4} \pm 2.8$ (< 32) [38]	$7.3^{+1.3}_{-1.2} \pm 1.3$ [48]	$15.1^{+3.4+1.4+2.0}_{-3.3-1.5-2.1}$ [62]	9.0 ± 2.3 ($S = 1.41$)
	-	-	$0.18 \pm 0.12 \pm 0.08$ [50]	$0.22^{+0.22+0.06}_{-0.23-0.02}$ [62]	0.19 ± 0.12
	$\rho^0 K^0$	< 39 [58]	-	< 12.4 [56]	< 12.4
	ωK^0	$10.0^{+5.4}_{-4.2} \pm 1.4$ (< 21) [38]	$5.3^{+1.4}_{-1.2} \pm 0.5$ [44]	$4.0^{+1.9}_{-1.6} \pm 0.5$ [45]	5.2 ± 1.1
	ϕK^0	$5.4^{+3.7}_{-2.7} \pm 0.7$ (< 12.3) [59]	$7.6^{+1.3}_{-1.2} \pm 0.5$ [47]	$9.0^{+2.2}_{-1.8} \pm 0.7$ [60]	7.8 ± 1.1

^aValue utilized in order to stabilize fits. See text.

$\lambda/(1 - \frac{\lambda^2}{2}) = 0.230$ and $\lambda = 0.224$ [66]. Thus the weak phase of p_P in the present convention is zero, while its strong phase is π . When we assume that $p'_V/p'_P = p_V/p_P = -1$, as suggested in Ref. [31] and as done in Refs. [10] and [12], we will have one free parameter $|p'_P|$. More generally, we shall consider fits with this ratio real or complex, adding one or two new parameters c and ϕ defined by $p'_V/p'_P = p_V/p_P = -c$ or $p'_V/p'_P = p_V/p_P = -ce^{i\phi}$. We do not introduce SU(3) breaking in penguin amplitudes.

- The magnitudes $|t_{P,V}|$ of the tree amplitudes and strong relative phases $\delta_{P,V}$ between them and the corresponding penguin amplitudes $p_{P,V}$ are free parameters: four in all. We relate strangeness-changing tree amplitudes to those for $\Delta S = 0$ using ratios of decay constants: $t'_V = \bar{\lambda}(f_K/f_\pi)t_V \simeq 0.281t_V$ and $t'_P = \bar{\lambda}(f_{K^*}/f_\rho)t_P \simeq 0.240t_P$. We thus assume the same relative strong phases between tree and penguin amplitudes in $\Delta S = 0$ and $|\Delta S| = 1$ processes. We use the following values of the decay constants [12, 36]: $f_\pi = 130.7$ MeV, $f_K = 159.8$ MeV, $f_\rho = 208$ MeV, $f_{K^*} = 217$ MeV.
- The weak phase γ of t_V and t_P is a free parameter. We assume it to be the same for both tree amplitudes.

- We take the electroweak penguin amplitude $P'_{EW,P}$ to have the same strong and weak phases as p'_P . Then the electroweak penguin contribution to s'_P , $-\frac{1}{3}P'_{EW,P}$, interferes destructively with p'_P , as was anticipated by explicit calculations [67]. Thus $-s'_P/p'_P$ is one real positive parameter. We ignore any contribution from the singlet penguin S'_P , which we expect to involve gluonic coupling to SU(3) flavor-singlet components of vector mesons and thus to be suppressed by the Okubo-Zweig-Iizuka (OZI) rule. We did not find a stable fit if we allowed the strong phase of the EWP contribution to s'_P to vary. The contribution $-\frac{1}{3}P'_{EW,P}$ appearing in s'_P then implies a corresponding contribution $+P'_{EW,P}$ in c'_P .
- The term c_P appears to play a key role in accounting for the deviation of the $B^+ \rightarrow \rho^0\pi^+$ and $B^+ \rightarrow \omega\pi^+$ decay rates from the predictions based on t_V alone. It contains two terms: a term C_P which we choose to have the same weak and strong phases as t_V , and a small EWP contribution $P_{EW,P} = -\bar{\lambda}P'_{EW,P}$ associated with the term taken in s'_P above. Thus C_P/t_V will be one additional real parameter, which will turn out to be positive in all our fits.
- We include a contribution from the amplitude C_V ,

TABLE III: Summary of predicted contributions to $\Delta S = 0$ decays of B mesons to one vector and one pseudoscalar mesons. Amplitude magnitudes $|A_{\text{exp}}|$ extracted from experiments are quoted in units of eV. The results are based on a fit with $p'_V = -p'_P$ and $\gamma \simeq 65^\circ$ (see Table V).

	Mode	Amplitudes	$ T ^a$	$ P ^a$	p_c (GeV)	$ A_{\text{exp}} ^b$	A_{CP}
$B^+ \rightarrow$	$\bar{K}^{*0} K^+$	p_P	0	7.5	2.539	< 24.1	-
	$K^{*+} \bar{K}^0$	p_V	0	7.5	2.539	-	-
	$\rho^0 \pi^+$	$-\frac{1}{\sqrt{2}}(t_V + c_P + p_V - p_P)$	25.1	10.6	2.582	31.4 ± 1.9	-0.17 ± 0.11
	$\rho^+ \pi^0$	$-\frac{1}{\sqrt{2}}(t_P + c_V + p_P - p_V)$	39.7	14.4	2.582	34.5 ± 4.2	0.23 ± 0.17
	$\rho^+ \eta$	$-\frac{1}{\sqrt{3}}(t_P + c_V + p_P + p_V + s_V)$	32.4	2.1 ^c	2.554	31.2 ± 4.7	0.06 ± 0.29
	$\rho^+ \eta'$	$\frac{1}{\sqrt{6}}(t_P + c_V + p_P + p_V + 4s_V)$	22.9	0.7 ^c	2.493	38.7 ± 6.6	-
	$\omega \pi^+$	$\frac{1}{\sqrt{2}}(t_V + c_P + p_P + p_V + 2s_P)$	25.1	0.006 ^d	2.580	25.3 ± 2.3	0.11 ± 0.21
	$\phi \pi^+$	s_P	0	0.009 ^d	2.539	< 6.7	-
$B^0 \rightarrow$	$\bar{K}^{*0} K^0$	p_P	0	7.5	2.539	-	-
	$K^{*0} \bar{K}^0$	p_V	0	7.5	2.539	-	-
	$\rho^- \pi^+$	$-(t_V + p_V)$	30.3	7.5	2.582	34.4 ± 3.4 ^e	-0.54 ± 0.19
	$\rho^+ \pi^-$	$-(t_P + p_P)$	43.1	7.5	2.582	40.1 ± 3.2 ^e	-0.16 ± 0.15
	$\rho^0 \pi^0$	$\frac{1}{2}(-c_P - c_V + p_P + p_V)$	9.2 ^f	2.7	2.582	< 17.1	-
	$\rho^0 \eta$	$\frac{1}{\sqrt{6}}(c_P - c_V - p_P - p_V - s_V)$	3.2 ^f	1.5 ^c	2.554	< 25.4	-
	$\rho^0 \eta'$	$\frac{1}{2\sqrt{3}}(c_V - c_P + p_P + p_V + 4s_V)$	2.3 ^f	0.5 ^c	2.493	< 38.0	-
	$\omega \pi^0$	$\frac{1}{2}(c_P - c_V + p_P + p_V + 2s_P)$	4.0 ^f	2.7 ^d	2.580	< 14.9	-
	$\omega \eta$	$-\frac{1}{\sqrt{6}}(c_P + c_V + p_P + p_V + 2s_P + s_V)$	7.5 ^f	1.5 ^{c,d}	2.552	< 37.6	-
	$\omega \eta'$	$\frac{1}{2\sqrt{3}}(c_P + c_V + p_P + p_V + 2s_P + 4s_V)$	5.3 ^f	0.5 ^{c,d}	2.491	< 85.1	-
	$\phi \pi^0$	$\frac{1}{\sqrt{2}}s_P$	0	0.006 ^d	2.539	< 24.3	-
	$\phi \eta$	$-\frac{1}{\sqrt{3}}s_P$	0	0.005 ^d	2.511	< 32.8	-
	$\phi \eta'$	$\frac{1}{\sqrt{6}}s_P$	0	0.004 ^d	2.447	< 11.1	-

^a T is the sum of all tree and color-suppressed amplitudes that contribute to a process.

P is the sum of all penguin amplitudes, including electroweak ones.

^b $|A_{\text{exp}}|$ is defined by Eq. (2) as an amplitude related to a CP -averaged branching ratio quoted in Table I.

^c No S_V contribution included. ^d No S_P contribution included.

^e Based on CP -averaged branching ratios quoted in Table I.

^f Takes account of the relative phase between C_P and C_V amplitudes.

motivated by the large $B^+ \rightarrow \rho^+ \eta$ and $B^+ \rightarrow \rho^+ \eta'$ branching ratios. We choose C_V to have the same strong and weak phases as t_P . We do not introduce $SU(3)$ breaking in C_P and C_V amplitudes and assume $C'_P = \bar{\lambda} C_P$, $C'_V = \bar{\lambda} C_V$.

- We take the electroweak penguin contribution to s'_V , $-\frac{1}{3}P'_{EW,V}$, to have the same strong and weak phases as p'_V . This contribution then implies corresponding contributions $+P'_{EW,V}$ in c'_V and $+P_{EW,V} = -\bar{\lambda}P'_{EW,V}$ in c_V . The apparent suppression of the decay $B^0 \rightarrow K^{*0} \pi^0$ (see Sec. VI) suggests that $P'_{EW,V}$ and p'_P are interfering destructively in this process, implying constructive interference in both charge states of $B \rightarrow K^* \eta$. We ignore any contribution from the singlet penguin S'_V in the absence of information about its magnitude and phase.

There are thus ten, eleven, or twelve parameters to fit 34 data points, depending on the assumption for the

p'_V/p'_P ratio: -1 , real, or complex. In fact, not all partial decay rates are independent, as we must have the following equalities between rate differences [68]:

$$\Gamma(B^0 \rightarrow \rho^- \pi^+) - \Gamma(\bar{B}^0 \rightarrow \rho^+ \pi^-) = (f_\pi/f_K)[\Gamma(\bar{B}^0 \rightarrow \rho^+ K^-) - \Gamma(B^0 \rightarrow \rho^- K^+)], \quad (4)$$

$$\Gamma(B^0 \rightarrow \rho^+ \pi^-) - \Gamma(\bar{B}^0 \rightarrow \rho^- \pi^+) = (f_\rho/f_{K^*})[\Gamma(\bar{B}^0 \rightarrow K^{*-} \pi^+) - \Gamma(B^0 \rightarrow K^{*+} \pi^-)]. \quad (5)$$

When transcribed into relations among branching ratios, these read, respectively,

$$(11.1 \pm 3.8) \times 10^{-6} \stackrel{?}{=} (2.8 \pm 1.9) \times 10^{-6}, \quad (6)$$

$$(4.4 \pm 4.1) \times 10^{-6} \stackrel{?}{=} (7.6 \pm 10.4) \times 10^{-6}. \quad (7)$$

The first of these is violated at the 2σ level. One reaps little gain in relaxing the assumption $p'_V = -p'_P$ since these relations are expected to hold regardless of p'_V/p'_P .

TABLE IV: Same as Table III for $|\Delta S| = 1$ decays of B mesons.

	Mode	Amplitudes	$ T' $	$ \mathcal{P}' $	p_c (GeV)	$ A_{\text{exp}} $	A_{CP}
$B^+ \rightarrow$	$K^{*0}\pi^+$	p'_P	0	32.6	2.561	31.2 ± 2.4	-
	$K^{*+}\pi^0$	$-\frac{1}{\sqrt{2}}(t'_P + c'_V + p'_P)$	9.4	39.7	2.562	< 58.1	-
	$K^{*+}\eta$	$-\frac{1}{\sqrt{3}}(t'_P + c'_V + p'_P - p'_V + s'_V)$	7.7	46.7 ^a	2.534	53.4 ± 3.5	0.10 ± 0.12
	$K^{*+}\eta'$	$\frac{1}{\sqrt{6}}(t'_P + c'_V + p'_P + 2p'_V + 4s'_V)$	5.4	16.5 ^a	2.472	< 62.8	-
	$\rho^0 K^+$	$-\frac{1}{\sqrt{2}}(t'_V + c'_P + p'_V)$	6.9	22.9	2.559	21.2 ± 2.1	-
	$\rho^+ K^0$	p'_V	0	32.6	2.559	< 72.3	-
	ωK^+	$\frac{1}{\sqrt{2}}(t'_V + c'_P + p'_V + 2s'_P)$	6.9	23.0 ^b	2.557	24.3 ± 1.8	-0.003 ± 0.122
	ϕK^+	$p'_P + s'_P$	0	32.5 ^b	2.516	31.6 ± 1.6	0.030 ± 0.072
$B^0 \rightarrow$	$K^{*+}\pi^-$	$-(t'_P + p'_P)$	10.3	32.6	2.562	42.4 ± 5.2	0.26 ± 0.35
	$K^{*0}\pi^0$	$-\frac{1}{\sqrt{2}}(c'_V - p'_P)$	2.1	6.3	2.562	< 20.3	-
	$K^{*0}\eta$	$-\frac{1}{\sqrt{3}}(c'_V + p'_P - p'_V + s'_V)$	1.7	46.7 ^a	2.534	46.0 ± 2.6	0.05 ± 0.10
	$K^{*0}\eta'$	$\frac{1}{\sqrt{6}}(c'_V + p'_P + 2p'_V + 4s'_V)$	1.2	16.5 ^a	2.471	< 39.8	-
	$\rho^- K^+$	$-(t'_V + p'_V)$	8.5	32.6	2.560	32.4 ± 4.2	0.19 ± 0.12
	$\rho^0 K^0$	$\frac{1}{\sqrt{2}}(p'_V - c'_P)$	0.9	23.1	2.559	< 38.2	-
	ωK^0	$\frac{1}{\sqrt{2}}(c'_P + p'_V + 2s'_P)$	0.9	23.0 ^b	2.557	24.6 ± 2.6	-
	ϕK^0	$p'_P + s'_P$	0	32.5 ^b	2.516	30.5 ± 2.1	-

^a No S'_V contribution included. ^b No S'_P contribution included.

When we assume $p'_V = -p'_P$, the specific expressions entering our fits include

$$A(B^+ \rightarrow K^{*0}\pi^+) = |p'_P| \quad (8)$$

$$A(B^0 \rightarrow \rho^- \pi^+) = -\bar{\lambda}|p'_P| - |t_V|e^{i(\delta_V + \gamma)} \quad (9)$$

$$A(B^0 \rightarrow \rho^- K^+) = |p'_P| - \bar{\lambda}\frac{f_K}{f_\pi}|t_V|e^{i(\delta_V + \gamma)} \quad (10)$$

$$A(B^0 \rightarrow \rho^+ \pi^-) = \bar{\lambda}|p'_P| + |t_P|e^{i(\delta_P + \gamma)} \quad (11)$$

$$A(B^0 \rightarrow K^{*+}\pi^-) = -|p'_P| + \bar{\lambda}\frac{f_{K^*}}{f_\rho}|t_P|e^{i(\delta_P + \gamma)} \quad (12)$$

The phase convention is such that $\delta_{V,P} = 0$ corresponds to $t_{P,V}$ having a phase of γ with respect to $p_{P,V}$. Amplitudes associated with the charge-conjugate modes can be obtained by flipping the sign of γ in the above expressions. The expressions lead to the rate relations (4) and (5) if one squares them and takes appropriate differences.

We now discuss two additional parameters in $(B^0, \bar{B}^0) \rightarrow \rho^\pm \pi^\mp$ which provide further constraints. These are measured in a time-dependent study by the BaBar Collaboration [50]:

$$S_{\rho\pi} = -0.13 \pm 0.18 \pm 0.04, \quad (13)$$

$$\Delta S_{\rho\pi} = 0.33 \pm 0.18 \pm 0.03, \quad (14)$$

which are related to the parameters S_{+-} and S_{-+} by

$$S_{\rho\pi} = (S_{+-} + S_{-+})/2, \quad (15)$$

$$\Delta S_{\rho\pi} = (S_{+-} - S_{-+})/2, \quad (16)$$

where

$$S_{+-} \equiv \frac{2\text{Im}\lambda^{+-}}{1 + |\lambda^{+-}|^2}, \quad (17)$$

$$S_{-+} \equiv \frac{2\text{Im}\lambda^{-+}}{1 + |\lambda^{-+}|^2}, \quad (18)$$

$$\lambda^{+-} \equiv \frac{q}{p} \frac{A(\bar{B}^0 \rightarrow \rho^+ \pi^-)}{A(B^0 \rightarrow \rho^+ \pi^-)}, \quad (19)$$

$$\lambda^{++} \equiv \frac{q}{p} \frac{A(\bar{B}^0 \rightarrow \rho^+ \pi^+)}{A(B^0 \rightarrow \rho^+ \pi^+)}, \quad (20)$$

$$\lambda^{-+} \equiv \frac{q}{p} \frac{A(\bar{B}^0 \rightarrow \rho^- \pi^+)}{A(B^0 \rightarrow \rho^- \pi^+)} \quad (21)$$

and $q/p = e^{-2i\beta}$ with $\beta = 23.7^\circ$. Since our fits predict the phases and magnitudes of all the relevant decay amplitudes, it is a simple matter to calculate the S 's. They provide crucial information on the relative strong phases of t_V and t_P , among other things. As mentioned, other observables $C_{\rho\pi}$, $\Delta C_{\rho\pi}$, and $A_{\rho\pi}$ as defined in Refs. [29] and [48] are related to information we already use in our fits and need not be considered separately. (See the Appendix.)

To see explicitly the constraints provided by $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$ it is helpful to calculate them in the limit in which the small penguin contributions to the $B \rightarrow \rho\pi$ amplitudes can be neglected. Defining $r \equiv |t_V/t_P|$ and $\delta \equiv \text{Arg}(t_V/t_P)$, one finds $S_{\pm\mp} = 2r \sin(2\alpha \pm \delta)/(1 + r^2)$ and

$$S_{\rho\pi} = \frac{2r}{1 + r^2} \sin 2\alpha \cos \delta, \quad \Delta S_{\rho\pi} = \frac{2r}{1 + r^2} \cos 2\alpha \sin \delta. \quad (22)$$

Both these quantities are small experimentally, consistent with solutions in which $\alpha = \pi - \beta - \gamma$ is near $\pi/2$ while δ is near zero or π . The non-zero central value of $\Delta S_{\rho\pi}$, in our favored solution to be discussed below, combines with a value of $\cos 2\alpha$ near -1 and other constraints to favor a small negative value of δ .

V. TESTS OF THE $p'_V = -p'_P$ ASSUMPTION

In this section, we note that the relation $p'_V = -p'_P$, proposed in Ref. [31] and used in previous discussions [10, 12], can be tested experimentally, and discuss the status of such tests.

As described before, information on $|p'_P|$ is directly obtained from the $B^+ \rightarrow K^{*0}\pi^+$ decay rate, which is predicted to be equal to that of its CP conjugate. (The absence of a CP asymmetry in this mode is one test of the present picture.) In principle, one can also extract $|p'_V|$ directly from the $B^+ \rightarrow \rho^+K^0$ rate (for which one also expects a vanishing CP asymmetry). Currently, only the CLEO group has reported an upper bound on the averaged branching ratio, $\overline{\mathcal{B}}(\rho^+K^0) < 48 \times 10^{-6}$ based on a sample of 2.6 million $B\bar{B}$ pairs. The present BaBar and Belle sample of approximately 100 times as much data would enable this mode to be observed at the predicted branching ratio of $\overline{\mathcal{B}}(\rho^+K^0) \simeq 12.6 \times 10^{-6}$ with good significance.

An indirect way to test $p'_P = -p'_V$ is to compare the ωK^0 and ϕK^0 modes. Using $c'_P = C'_P + P'_{EW,P}$ and $s'_P = S'_P - P'_{EW,P}/3$ [Eq. (1)] and the facts that C'_P is smaller than $P'_{EW,P}$ by a factor of about 0.22 (see Table I in Ref. [12]) and that S'_P is OZI-suppressed, we can safely neglect them and have

$$A(\omega K^0) = \frac{1}{\sqrt{2}} \left(p'_V + \frac{1}{3} P'_{EW,P} \right), \quad (23)$$

$$A(\phi K^0) = p'_P - \frac{1}{3} P'_{EW,P}. \quad (24)$$

Therefore, the amplitude magnitudes are related by a factor of $\sqrt{2}$ if $p'_P = -p'_V$. Current data, using the average for $B^+ \rightarrow \phi K^+$ and $B^0 \rightarrow \phi K^0$ amplitudes (expected to be equal within our approximations) show that $\sqrt{2}|A(B^0 \rightarrow \omega K^0)/A(B \rightarrow \phi K)| = 1.13 \pm 0.13$, consistent with 1. [Note added: The updated branching ratio $\mathcal{B}(B^0 \rightarrow \phi K^0)$ [47] results in a slightly smaller ratio of amplitudes: $\sqrt{2}|A(B^0 \rightarrow \omega K^0)/A(B \rightarrow \phi K)| = 1.11 \pm 0.13$.]

Observation of the $\rho^\pm K^0$ mode and more precise determination of ωK^0 and ϕK^0 modes thus will be very helpful in justifying the assumption of $p'_P = -p'_V$. However, we find in the next section that global fits to data in which this assumption is relaxed are not very different from those in which $p'_P = -p'_V$ is assumed.

VI. DISCUSSION OF PREDICTIONS

Plots of χ^2 as a function of γ for three fits are shown in Fig. 1. Three local minima are found, around $\gamma = 26^\circ$, 63° , and 162° . The fit with p'_V/p'_P real gives a χ^2 very similar to that with $p'_V/p'_P = -1$ for $\gamma \simeq 26^\circ$ and to that with p'_V/p'_P complex for the other two minima, so we shall not consider it further. The magnitudes of individual amplitudes and the strong phases determined in the fits with $p'_V/p'_P = -1$ and p'_V/p'_P complex are compared with one another for the three local minima in Table V. The corresponding predictions of these fits are compared with one another and with experiment in Tables VI and VII.

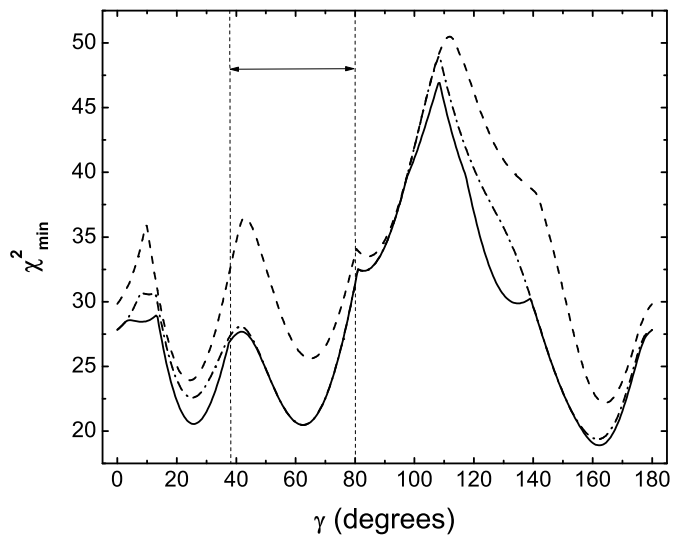


FIG. 1: $(\chi^2)_{\min}$, obtained by minimizing over all remaining fit parameters, as a function of the weak phase γ . Dash-dotted curve: $p'_V/p'_P = -1$ (24 d.o.f.); dashed curve: p'_V/p'_P real (23 d.o.f.); solid curve: p'_V/p'_P complex (22 d.o.f.).

In the absence of information on $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$, Fig. 1 would be symmetric under $\gamma \rightarrow \pi - \gamma$, since all other observables would be unchanged under a simultaneous change of strong phases $\delta_{P,V} \rightarrow \pi - \delta_{P,V}$. The time-dependent CP asymmetries in $B \rightarrow \rho^\mp \pi^\pm$ break this symmetry. Nonetheless, considerable ambiguity remains, and it is necessary to appeal both to theory and to other experiments to resolve it.

We shall concentrate upon solutions consistent with γ in the range allowed by fits [69] to other observables, $38^\circ < \gamma < 80^\circ$ at the 95% confidence level. Solutions outside this range not only conflict with these fits, but have large final-state phases (mod π) which are unlikely in QCD factorization [24]. We shall point out ways in which a distinction among solutions can be made experimentally. Magnitudes and phases of the dominant invariant amplitudes in the solution with $\gamma \simeq 63^\circ$ and complex p'_V/p'_P are shown in Fig. 2. Using this figure and Tables

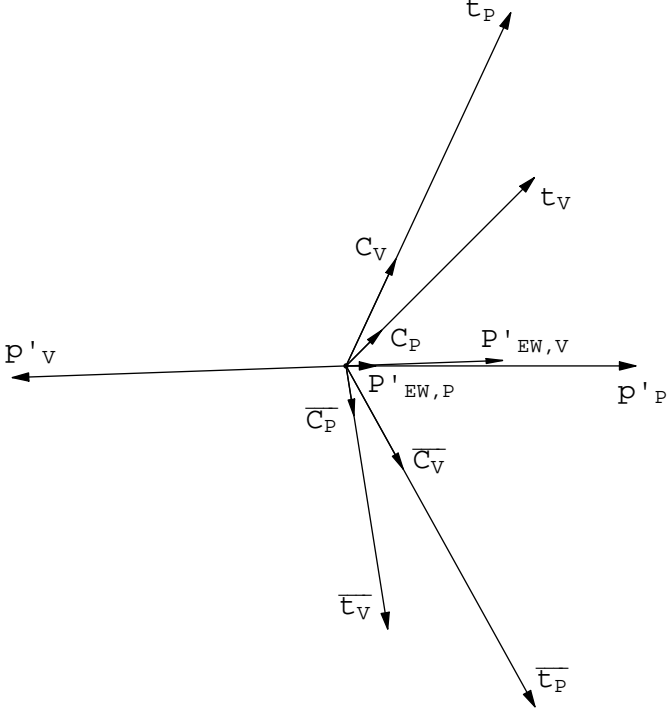


FIG. 2: Magnitudes and phases of dominant invariant amplitudes in solution with $\gamma \simeq 63^\circ$ and complex p'_V/p'_P . Other amplitudes are given by $p_{(P,V)} = -0.230 p'_{(P,V)}$, $P'_{EW,(P,V)} = -0.230 P'_{EW,(P,V)}$, $t'_P = 0.240 t_P$, $t'_V = 0.281 t_V$, $C'_{(P,V)} = 0.230 C_{(P,V)}$.

III and IV one can see whether a given process involves constructive or destructive tree-penguin interference.

Errors for all solutions with complex p'_V/p'_P have been estimated using a Monte Carlo method in which parameter sets were generated leading to χ^2 values no more than 1 unit above the minimum, and the spread in predictions was studied. For any prediction depending on a single parameter, the error in that parameter was used to obtain the error in the prediction.

Within the range $38^\circ \leq \gamma \leq 80^\circ$ [69], the present fits specify γ to within a few degrees at the 1σ level. Values corresponding to $\Delta\chi^2 \leq 1$ above the minimum for $p'_V/p'_P = -1$ and complex are $\gamma = (65 \pm 6)^\circ$ and $(63 \pm 6)^\circ$, respectively. Ranges for $\Delta\chi^2 \leq 3.84$ above the minimum (95% c.l. limits) are 54° – 75° and 51° – 73° in the two fits.

The local minima found at $\gamma \simeq (26, 162)^\circ$ are associated with larger C'_V and $P'_{EW,V}$, leading to larger predicted branching ratios $\mathcal{B}(B^+ \rightarrow K^{*+}\pi^0) = (22.1^{+4.2}_{-5.1}, 18.2^{+5.0}_{-4.1}) \times 10^{-6}$ versus the $15.0^{+3.3}_{-2.8}$ prediction for $\gamma \simeq 63^\circ$. These predictions do not conflict with the present CLEO [38] bound of 31×10^{-6} and are not too far below it. The branching ratio of this decay should be measurable very soon. The solution for $\gamma \simeq 162^\circ$ also predicts a larger CP asymmetry for $B^+ \rightarrow \rho^+\pi^0$, closer to the present central value which is slightly disfavored in the fits with $\gamma \simeq 63^\circ$. The fits with $\gamma \simeq (26, 162)^\circ$ pre-

dict larger branching ratios and CP asymmetries for the color-suppressed all-neutral decay modes $B^0 \rightarrow \rho^0(\eta, \eta')$ and $B^0 \rightarrow \omega\pi^0$ than do the fits with $\gamma \simeq 63^\circ$.

Several observables provide the main contributions to χ^2 . These are summarized in Table VIII. The large CP asymmetry in $B^0 \rightarrow \rho^-\pi^+$ provides the largest $\Delta\chi^2$ in all three cases. The only other contributions with $\Delta\chi^2 \geq 1$ occur for $\gamma = 162^\circ$, with $\Delta\chi^2[A_{CP}(B^+ \rightarrow K^{*+}\eta)] = 1.3$, and for $\gamma = 26^\circ$, with $\Delta\chi^2[\mathcal{B}(B^0 \rightarrow \omega K^0)] = 1.3$.

We comment on several predictions of the fits with $\gamma \simeq 63^\circ$ which appear to be of general nature, not depending on specific assumptions about p'_V/p'_P or symmetry breaking.

All fits predict $|t_V| < |t_P|$. In a factorization picture t_V involves the production of a π^\pm ($f_\pi = 130.7$ MeV) by the weak current, whereas t_P involves production of a ρ^\pm ($f_\rho = 208$ MeV). This inequality is therefore not so surprising. One has $|t_V/t_P| \simeq 0.68$ while $f_\pi/f_\rho \simeq 0.63$, suggesting within factorization that the $B \rightarrow \rho$ and $B \rightarrow \pi$ form factors are similar. The values of $|t_V|$ we find are comparable to that of $|t|$, the tree amplitude in $B \rightarrow PP$, which was found in [13] to be $|t| = 27.1 \pm 3.9$ eV. One also finds $|t_P/t| \simeq f_\rho/f_\pi$, as expected from factorization when the ρ - π mass difference is neglected.

All fits predict $p'_V \simeq -p'_P$, even if this equality is not enforced. This is largely due to the need for constructive interference between p'_V and $-p'_P$ in the decays $B \rightarrow K^*\eta$, as proposed in Ref. [31]. We find $|p'_P/p'| \simeq 0.7$, where p' is the strangeness-changing penguin amplitude in $B \rightarrow PP$ decays, with magnitude [13] $|p'| = 45.7 \pm 1.7$ eV determined primarily by the decay $B^+ \rightarrow K^0\pi^+$.

The ratio $-s'_P/p'_P$, corresponding to the electroweak penguin contribution to s'_P , is found to be 0.03 ± 0.04 for the $\gamma \simeq 63^\circ$ fit, somewhat smaller than expectations [67]. The central value of the ratio C_P/t_V is about 0.2. These amplitudes interfere constructively in such processes as $B^+ \rightarrow \rho^0\pi^+$ and $B^+ \rightarrow \omega\pi^+$. The corresponding ratio for C_V/t_P is slightly larger, about 0.3, helping to enhance the predicted rates for $B^+ \rightarrow \rho^+(\pi^0, \eta, \eta')$. A similar constructive interference between C and T terms appears to account for an enhancement of the $B^+ \rightarrow \pi^+\pi^0$ decay (see, e.g., [18]).

Small strong phases (mod π) are favored, implying that the relative strong phase between t_V and t_P is small. This is consistent with the prediction of QCD factorization methods [24].

The pattern of strong and weak phases obtained here is such that there is a small amount of constructive tree-penguin interference in the CP -averaged branching ratios for $B^0 \rightarrow \rho^-\pi^+$ and $B^0 \rightarrow K^{*+}\pi^-$, and a small amount of destructive interference in $B^0 \rightarrow \rho^+\pi^-$ and $B^0 \rightarrow \rho^-K^+$. The preference of the fits for large values of γ (small values of $\cos\gamma$), when final-state phases are small mod π , is due in part to the fact that these interference effects are relatively small.

We shall comment below on details which depend on specific assumptions. First we discuss some specific decay modes for which predictions are fairly stable over the

TABLE V: Comparison of parameters extracted in fits to branching ratios and CP asymmetries under various assumptions. Values of the topological amplitudes are quoted in units of eV. Probabilities are those for χ^2 to exceed the value shown for the indicated number of degrees of freedom.

Quantity	Value in fit					
	$ \leftarrow$	$p'_V/p'_P = -1$	$\rightarrow $	$ \leftarrow$	$p'_V/p'_P = -ce^{i\phi}$	$\rightarrow $
γ	$(24 \pm 5)^\circ$	$(65 \pm 6)^\circ$	$(164 \pm 5)^\circ$	$(26 \pm 5)^\circ$	$(63 \pm 6)^\circ$	$(162^{+5}_{-6})^\circ$
ϕ	0 (input)	0 (input)	0 (input)	$(35^{+12}_{-21})^\circ$	$(2 \pm 18)^\circ$	$(-20^{+31}_{-23})^\circ$
$ p'_P $	$32.4^{+1.4}_{-1.5}$	32.6 ± 1.5	$32.4^{+1.4}_{-1.5}$	$32.5^{+1.7}_{-1.6}$	$32.2^{+1.4}_{-1.5}$	32.0 ± 1.6
$ p'_V $	$32.4^{+1.4}_{-1.5}$	32.6 ± 1.5	$32.4^{+1.4}_{-1.5}$	$31.3^{+3.6}_{-3.0}$	$37.1^{+2.3}_{-2.4}$	$35.3^{+3.4}_{-3.6}$
$ s'_P ^a$	0.9 ± 1.3	0.0 ± 1.3	0.5 ± 1.3	$1.7^{+1.8}_{-1.6}$	1.1 ± 1.3	$1.1^{+1.7}_{-1.5}$
$ s'_V ^a$	$6.2^{+2.2}_{-2.1}$	$7.9^{+2.4}_{-2.3}$	$6.9^{+2.3}_{-2.2}$	$8.5^{+2.8}_{-3.5}$	$5.8^{+2.4}_{-2.1}$	$6.1^{+3.8}_{-2.9}$
$ t_P $	$46.7^{+3.1}_{-3.4}$	$43.1^{+3.2}_{-3.4}$	$46.1^{+3.2}_{-3.4}$	$46.2^{+3.2}_{-3.4}$	$43.3^{+3.2}_{-3.4}$	$47.0^{+3.1}_{-3.4}$
$ t_V $	$32.5^{+3.6}_{-3.9}$	$30.3^{+3.4}_{-3.7}$	$31.9^{+3.6}_{-4.0}$	$33.8^{+3.7}_{-4.1}$	$29.6^{+3.4}_{-3.7}$	$30.9^{+4.2}_{-4.7}$
δ_P	$(184 \pm 12)^\circ$	$(181 \pm 8)^\circ$	$(36 \pm 13)^\circ$	$(199 \pm 14)^\circ$	$(182 \pm 14)^\circ$	$(16^{+20}_{-23})^\circ$
δ_V	$(-87 \pm 11)^\circ$	$(-18^{+7}_{-8})^\circ$	$(-100^{+10}_{-12})^\circ$	$(-102^{+19}_{-14})^\circ$	$(-20^{+9}_{-10})^\circ$	$(-105^{+19}_{-26})^\circ$
$ C_P $	$6.0^{+4.3}_{-4.1}$	$5.3^{+4.3}_{-4.0}$	$5.7^{+4.4}_{-4.1}$	$6.8^{+4.5}_{-4.2}$	$5.6^{+4.2}_{-4.0}$	$6.0^{+4.4}_{-4.1}$
$ C_V $	$16.8^{+5.5}_{-5.7}$	$13.1^{+5.7}_{-6.0}$	$15.0^{+5.6}_{-5.8}$	$17.3^{+5.6}_{-5.8}$	$13.1^{+5.7}_{-6.0}$	$14.9^{+5.6}_{-5.8}$
Fit properties:						
$\chi^2/\text{d.f.}$	23.9/24	25.5/24	22.1/24	20.6/22	20.5/22	18.9/22
% c.l.	47%	38%	57%	55%	55%	65%
Derived quantities:						
$ p'_V/p'_P $	1 (input)	1 (input)	1 (input)	$0.96^{+0.12}_{-0.10}$	1.15 ± 0.07	1.10 ± 0.12
$-s'_P/p'_P$	0.03 ± 0.04	0.00 ± 0.04	$0.02^{+0.04}_{-0.05}$	$0.05^{+0.06}_{-0.05}$	0.03 ± 0.04	0.04 ± 0.05
s'_V/p'_V	$0.19^{+0.08}_{-0.07}$	$0.24^{+0.09}_{-0.08}$	$0.21^{+0.08}_{-0.07}$	$0.27^{+0.13}_{-0.12}$	$0.16^{+0.08}_{-0.06}$	$0.17^{+0.14}_{-0.09}$
$ t_V/t_P $	0.70 ± 0.10	0.70 ± 0.10	$0.69^{+0.11}_{-0.10}$	0.73 ± 0.10	0.68 ± 0.10	0.66 ± 0.11
$\text{Arg}(t_V/t_P)$	$(-91 \pm 10)^\circ$	$(-19 \pm 9)^\circ$	$(43^{+12}_{-11})^\circ$	$(-87 \pm 10)^\circ$	$(-20 \pm 10)^\circ$	$(38^{+12}_{-11})^\circ$
C_P/t_V	$0.19^{+0.17}_{-0.13}$	$0.17^{+0.18}_{-0.14}$	$0.18^{+0.18}_{-0.14}$	$0.20^{+0.17}_{-0.13}$	$0.19^{+0.19}_{-0.14}$	$0.19^{+0.19}_{-0.14}$
C_V/t_P	$0.36^{+0.15}_{-0.13}$	$0.30^{+0.16}_{-0.15}$	$0.33^{+0.15}_{-0.14}$	$0.38^{+0.15}_{-0.14}$	$0.30^{+0.16}_{-0.15}$	$0.32^{+0.14}_{-0.13}$
$ p_P = 0.230 p'_P $	7.4 ± 0.3	7.5 ± 0.3	7.4 ± 0.3	7.5 ± 0.4	7.4 ± 0.3	7.4 ± 0.4
$ p_V = 0.230 p'_V $	7.4 ± 0.3	7.5 ± 0.3	7.4 ± 0.3	$7.2^{+0.8}_{-0.7}$	$8.5^{+0.5}_{-0.6}$	8.1 ± 0.8
$ s_P = 0.230 s'_P $	0.2 ± 0.3	0.0 ± 0.3	0.1 ± 0.3	0.4 ± 0.4	0.3 ± 0.3	$0.3^{+0.4}_{-0.3}$
$ s_V = 0.230 s'_V $	1.4 ± 0.5	$1.8^{+0.6}_{-0.5}$	1.6 ± 0.5	$2.0^{+0.6}_{-0.8}$	$1.3^{+0.6}_{-0.5}$	$1.4^{+0.9}_{-0.7}$
$ t'_V = 0.281 t_V $	$9.1^{+1.0}_{-1.1}$	8.5 ± 1.0	$9.0^{+1.0}_{-1.1}$	$9.5^{+1.0}_{-1.2}$	8.3 ± 1.0	$8.7^{+1.2}_{-1.3}$
$ t'_P = 0.240 t_P $	$11.2^{+0.7}_{-0.8}$	10.3 ± 0.8	11.0 ± 0.8	11.1 ± 0.8	10.4 ± 0.8	$11.3^{+0.7}_{-0.8}$
$ C'_P = 0.230 C_P $	$1.4^{+1.0}_{-0.9}$	$1.2^{+1.0}_{-0.9}$	$1.3^{+1.0}_{-0.9}$	1.6 ± 1.0	$1.3^{+1.0}_{-0.9}$	$1.4^{+1.0}_{-0.9}$
$ C'_V = 0.230 C_V $	3.8 ± 1.3	$3.0^{+1.3}_{-1.4}$	3.4 ± 1.3	4.0 ± 1.3	$3.0^{+1.3}_{-1.4}$	3.4 ± 1.3

^aEWP contribution only.

range of assumptions.

A. $\Delta S = 0$ decays

The decay $B^+ \rightarrow \bar{K}^{*0} K^+$ is predicted to be dominated by the p_P amplitude and thus to have zero CP asymmetry. Its branching ratio is expected to be about 0.5×10^{-6} , considerably below present upper limits. Any deviations from these predictions could indicate the importance of an annihilation amplitude or, equivalently, important rescattering effects. One expects a similar or slightly larger prediction for $B^+ \rightarrow K^{*+} \bar{K}^0$.

A non-zero negative CP asymmetry is predicted for

$B^+ \rightarrow \rho^0 \pi^+$, as a result of the interference of the amplitudes $t_V + c_P$ and $p_V - p_P$. In our favored solution (with $\gamma \simeq 63^\circ$) this arises as the result of a small but non-negligible relative final-state phase δ_V . The same phase contributes a CP asymmetry of opposite sign to $B^+ \rightarrow \rho^0 K^+$ and $B^+ \rightarrow \omega K^+$, as we shall see below. It appears to be generated in our fit by the appreciable CP asymmetry in $B^0 \rightarrow \rho^- \pi^+$, and also leads to a non-zero prediction for $\Delta S_{\rho\pi}$.

The decay $B^+ \rightarrow \rho^+ \pi^0$ is expected to have a branching ratio of about 12×10^{-6} , consistent with the recently reported level [39].

The decay $B^+ \rightarrow \phi \pi^+$, dominated by an amplitude coming from the electroweak penguin in s_P , is included

TABLE VI: Comparison of predicted and experimental branching ratios and CP asymmetries A_{CP} for some $\Delta S = 0$ decays of B mesons. References are given in Table I. Predictions are shown only for the fit with p'_V/p'_P complex.

Mode	\leftarrow	\mathcal{B} (units of 10^{-6})			\rightarrow	\leftarrow	A_{CP}			\rightarrow
		Prediction			Expt.		Prediction			Expt.
	$\gamma = 26^\circ$	$\gamma = 63^\circ$	$\gamma = 162^\circ$			$\gamma = 26^\circ$	$\gamma = 63^\circ$	$\gamma = 162^\circ$		
$B^+ \rightarrow$	$\overline{K}^{*0} K^+$	$0.51^{+0.06}_{-0.05}$	0.50 ± 0.05	0.49 ± 0.05	< 5.3	0	0	0		
	$K^{*+} \bar{K}^0$	$0.47^{+0.12}_{-0.09}$	0.66 ± 0.08	0.60 ± 0.12		0	0	0		
	$\rho^0 \pi^+$	$8.7^{+0.3}_{-0.4}$	9.0 ± 0.4	$9.2^{+0.5}_{-0.6}$	9.1 ± 1.1	-0.24 ± 0.04	-0.16 ± 0.04	-0.15 ± 0.03	-0.17 ± 0.11	
	$\rho^+ \pi^0$	$10.1^{+1.6}_{-1.7}$	$11.8^{+1.6}_{-1.8}$	$10.2^{+1.6}_{-1.7}$	11.0 ± 2.7	$0.03^{+0.08}_{-0.10}$	-0.01 ± 0.06	$0.16^{+0.04}_{-0.05}$	0.23 ± 0.17	
	$\rho^+ \eta^a$	$11.1^{+1.3}_{-1.5}$	$9.5^{+1.3}_{-1.5}$	$10.7^{+1.4}_{-1.5}$	8.9 ± 2.7	$-0.05^{+0.03}_{-0.02}$	-0.01 ± 0.07	-0.01 ± 0.03	0.06 ± 0.29	
	$\rho^+ \eta'^a$	$6.4^{+0.7}_{-0.8}$	4.9 ± 0.7	$5.8^{+0.7}_{-0.8}$	13.3 ± 4.5	$-0.06^{+0.04}_{-0.03}$	-0.01 ± 0.08	-0.03 ± 0.04		
	$\omega \pi^+{}^b$	6.2 ± 0.5	5.9 ± 0.4	5.6 ± 0.7	5.9 ± 1.1	$0.01^{+0.04}_{-0.03}$	$-0.03^{+0.10}_{-0.13}$	$-0.03^{+0.02}_{-0.06}$	0.10 ± 0.21	
	$\phi \pi^+{}^b$	$0.001^{+0.005}_{-0.001}$	$0.001^{+0.002}_{-0.001}$	$0.001^{+0.003}_{-0.001}$	< 0.41	0	0	0		
$B^0 \rightarrow$	$\overline{K}^{*0} K^0$	0.47 ± 0.05	0.46 ± 0.04	0.46 ± 0.05		0	0	0		
	$K^{*0} \bar{K}^0$	$0.44^{+0.11}_{-0.08}$	0.61 ± 0.08	0.55 ± 0.11		0	0	0		
	$\rho^- \pi^+$	9.5 ± 1.9	10.0 ± 2.0	$9.9^{+1.7}_{-1.8}$	10.2 ± 2.0	-0.19 ± 0.04	-0.13 ± 0.06	-0.13 ± 0.03	-0.54 ± 0.19	
	$\rho^+ \pi^-$	13.8 ± 2.2	14.0 ± 2.2	14.0 ± 2.1	13.8 ± 2.2	$-0.06^{+0.05}_{-0.04}$	-0.01 ± 0.08	$0.04^{+0.04}_{-0.05}$	-0.16 ± 0.15	
	$\rho^0 \pi^0$	$0.63^{+0.35}_{-0.23}$	$0.60^{+0.45}_{-0.32}$	$0.74^{+0.48}_{-0.34}$	< 2.5	$0.43^{+0.09}_{-0.16}$	$0.09^{+0.33}_{-0.31}$	$0.21^{+0.09}_{-0.13}$		
	$\rho^0 \eta^a$	$0.43^{+0.27}_{-0.20}$	$0.08^{+0.17}_{-0.08}$	$0.13^{+0.17}_{-0.09}$	< 5.5	$-0.20^{+0.11}_{-0.08}$	-0.09 ± 0.46	$-0.14^{+0.22}_{-0.14}$		
	$\rho^0 \eta'^a$	$0.35^{+0.17}_{-0.14}$	$0.07^{+0.10}_{-0.05}$	$0.13^{+0.11}_{-0.07}$	< 12	$-0.10^{+0.09}_{-0.05}$	$0.08^{+0.36}_{-0.30}$	$-0.11^{+0.16}_{-0.07}$		
	$\omega \pi^0{}^b$	$0.36^{+0.36}_{-0.23}$	$0.11^{+0.21}_{-0.06}$	$0.14^{+0.26}_{-0.12}$	< 1.9	$0.35^{+0.26}_{-0.35}$	$-0.26^{+0.71}_{-0.60}$	$0.41^{+0.56}_{-0.66}$		
	$\omega \eta^{a,b}$	$0.30^{+0.26}_{-0.19}$	$0.44^{+0.33}_{-0.25}$	$0.40^{+0.30}_{-0.22}$	< 12	$-0.13^{+0.13}_{-0.09}$	-0.00 ± 0.22	$-0.02^{+0.08}_{-0.11}$		
	$\omega \eta'^{a,b}$	$0.26^{+0.17}_{-0.13}$	$0.27^{+0.18}_{-0.14}$	$0.27^{+0.18}_{-0.14}$	< 60	$-0.23^{+0.13}_{-0.09}$	-0.04 ± 0.22	$-0.11^{+0.11}_{-0.10}$		
	$\phi \pi^0{}^b$	$0.001^{+0.002}_{-0.001}$	$0.000^{+0.001}_{-0.000}$	$0.000^{+0.002}_{-0.000}$	< 5	0	0	0		
	$\phi \eta^b$	< 0.001	< 0.001	< 0.001	< 9	0	0	0		
	$\phi \eta'^b$	< 0.001	< 0.001	< 0.001	< 1.0	0	0	0		
	$S_{\rho\pi}$					$-0.19^{+0.06}_{-0.07}$	$-0.18^{+0.21}_{-0.17}$	$-0.17^{+0.18}_{-0.14}$	-0.13 ± 0.18	
$\Delta S_{\rho\pi}$					$0.22^{+0.18}_{-0.17}$	$0.28^{+0.04}_{-0.05}$	$0.38^{+0.05}_{-0.08}$	0.33 ± 0.18		

^aNo S_V contribution included. ^bNo S_P contribution included.

for completeness. We do not expect it to be observed any time soon. The corresponding predicted rates for $B^0 \rightarrow \phi(\pi^0, \eta, \eta')$ are factors of (2,3,6) smaller, respectively.

The branching ratio for the newly observed decay $B^+ \rightarrow \rho^+ \eta$ [42] is well reproduced, in part because of the enhancement associated with the constructive interference between t_P and C_V . The large value of C_V is driven by the attempt to fit an even larger branching ratio for $B^+ \rightarrow \rho^+ \eta'$ reported in the same experiment. The penguin contributions are expected to cancel if $p_V = -p_P$, so both of these decays are expected to have zero or very small CP asymmetries. This stands in contrast to the large asymmetries expected for $B^+ \rightarrow \pi^+ \eta$ and $B^+ \rightarrow \pi^+ \eta'$ [13]. In $B^+ \rightarrow \rho^+ \eta^{(\prime)}$, the QCD penguin contributions associated with the $u\bar{u}$ and $d\bar{d}$ components of the $\eta^{(\prime)}$ nearly cancel one another if $p_V \simeq -p_P$, while in $B^+ \rightarrow \pi^+ \eta^{(\prime)}$, these contributions reinforce one another.

We predict $\mathcal{B}(B^+ \rightarrow \rho^+ \eta')/\mathcal{B}(B^+ \rightarrow \rho^+ \eta) \simeq 1/2$, whereas the observed ratio exceeds 1. This may reflect a shortcoming of our description of the η' wave function, which has been argued in Ref. [19] to contain important symmetry-breaking effects.

We are unable to accommodate the large central value

of the CP asymmetry in $B^0 \rightarrow \rho^- \pi^+$. This will be true of any formalism which respects the rate difference relation (4), which is seen in Eq. (6) to be poorly obeyed by central values.

B. $|\Delta S| = 1$ decays

The decay $B^+ \rightarrow K^{*0} \pi^+$, dominated by p'_P , is the main source of information on that amplitude. It is expected to have zero CP asymmetry. Better measurement of its branching ratio would reduce the errors on $|p'_P|$.

Electroweak penguin contributions play an important role in the large predicted value $\mathcal{B}(B^+ \rightarrow K^{*+} \pi^0) \simeq 15 \times 10^{-6}$. As mentioned, one may expect detection of this mode in the near future, and it may help to choose among various local χ^2 minima in Fig. 1.

The predictions for $B^+ \rightarrow K^{*+} \eta$ and $B^+ \rightarrow K^{*+} \eta'$ include a small tree contribution whereas no such contribution is expected for the corresponding decays $B^0 \rightarrow K^{*0} \eta$ and $B^0 \rightarrow K^{*0} \eta'$. This leads us to expect a slight enhancement of $\mathcal{B}(B^+ \rightarrow K^{*+} \eta)$ with respect to $\mathcal{B}(B^0 \rightarrow K^{*0} \eta)$, as suggested by the data. The successful predic-

TABLE VII: Same as Table VI for $|\Delta S| = 1$ decays of B mesons.

Mode	\leftarrow	\mathcal{B} (units of 10^{-6})			\rightarrow	\leftarrow	A_{CP}			\rightarrow
		Prediction			Expt.		Prediction			Expt.
	$\gamma = 26^\circ$	$\gamma = 63^\circ$	$\gamma = 162^\circ$			$\gamma = 26^\circ$	$\gamma = 63^\circ$	$\gamma = 162^\circ$		
$B^+ \rightarrow K^{*0}\pi^+$	$9.7^{+1.1}_{-0.9}$	9.5 ± 0.9	9.4 ± 1.0	9.0 ± 1.4		0	0	0		
$K^{*+}\pi^0$	$22.1^{+4.2}_{-5.1}$	$15.0^{+3.3}_{-2.8}$	$18.2^{+5.0}_{-4.1}$	< 31	0.01 ± 0.03	0.01 ± 0.05	$-0.04^{+0.03}_{-0.02}$			
$K^{*+}\eta^a$	$25.1^{+2.0}_{-0.9}$	$23.4^{+1.4}_{-1.2}$	$25.1^{+2.3}_{-0.9}$	25.9 ± 3.4	-0.00 ± 0.02	0.00 ± 0.02	-0.04 ± 0.02	0.10 ± 0.12		
$K^{*+}\eta'^a$	$2.2^{+2.1}_{-0.7}$	$2.8^{+1.2}_{-0.3}$	$2.4^{+3.1}_{-0.6}$	< 12	$0.26^{+0.04}_{-0.16}$	0.01 ± 0.16	$0.20^{+0.02}_{-0.14}$			
$\rho^0 K^+$	$4.8^{+1.6}_{-0.3}$	$4.4^{+0.8}_{-0.6}$	$4.5^{+1.5}_{-0.3}$	4.1 ± 0.8	0.24 ± 0.04	0.21 ± 0.10	$0.19^{+0.04}_{-0.05}$			
$\rho^+ K^0$	$9.0^{+2.2}_{-1.6}$	12.6 ± 1.6	$11.4^{+2.3}_{-2.2}$	< 48	0	0	0			
ωK^{+b}	$5.3^{+1.5}_{-0.3}$	$5.0^{+0.8}_{-0.4}$	$5.1^{+1.4}_{-0.3}$	5.4 ± 0.8	0.24 ± 0.04	0.19 ± 0.08	$0.18^{+0.04}_{-0.05}$	-0.003 ± 0.122		
ϕK^{+b}	8.6 ± 0.3	8.7 ± 0.4	$8.6^{+0.2}_{-0.3}$	9.0 ± 0.9	0	0	0	0.030 ± 0.072		
$B^0 \rightarrow K^{*+}\pi^-$	$15.3^{+1.3}_{-1.1}$	12.4 ± 0.9	$15.5^{+1.2}_{-1.1}$	15.3 ± 3.8	0.06 ± 0.04	0.01 ± 0.10	$-0.03^{+0.05}_{-0.04}$	0.26 ± 0.35		
$K^{*0}\pi^0$	$1.3^{+1.3}_{-0.7}$	$0.8^{+1.0}_{-0.6}$	$0.7^{+1.0}_{-0.3}$	< 3.5	$-0.19^{+0.06}_{-0.05}$	$-0.03^{+0.22}_{-0.29}$	$-0.03^{+0.13}_{-0.10}$			
$K^{*0}\eta^a$	$18.4^{+1.7}_{-0.7}$	$19.1^{+1.2}_{-1.1}$	$18.5^{+1.9}_{-0.9}$	17.8 ± 2.0	$-0.001^{+0.008}_{-0.006}$	0.001 ± 0.006	$-0.012^{+0.004}_{-0.003}$	0.05 ± 0.10		
$K^{*0}\eta'^a$	$2.8^{+2.1}_{-0.5}$	$3.0^{+1.2}_{-0.3}$	$2.9^{+2.8}_{-0.5}$	< 6.4	$0.05^{+0.02}_{-0.04}$	0.00 ± 0.03	$0.04^{+0.02}_{-0.03}$			
$\rho^- K^+$	$10.1^{+2.7}_{-0.9}$	$10.0^{+1.4}_{-1.3}$	$9.9^{+2.5}_{-1.1}$	9.0 ± 2.3	$0.21^{+0.04}_{-0.03}$	$0.16^{+0.06}_{-0.07}$	$0.15^{+0.03}_{-0.04}$	0.19 ± 0.12		
$\rho^0 K^0$	$5.3^{+1.8}_{-1.5}$	$7.2^{+2.1}_{-1.9}$	$6.4^{+1.6}_{-1.8}$	< 12.4	-0.04 ± 0.03	-0.02 ± 0.01	-0.02 ± 0.02			
ωK^{0b}	$3.9^{+1.1}_{-0.6}$	$5.3^{+0.8}_{-0.4}$	$4.9^{+1.1}_{-0.8}$	5.2 ± 1.1	0.04 ± 0.03	0.02 ± 0.02	0.02 ± 0.02			
ϕK^{0b}	$7.9^{+0.2}_{-0.3}$	8.1 ± 0.3	$8.0^{+0.2}_{-0.3}$	7.8 ± 1.1	0	0	0			

^aNo S'_V contribution included. ^bNo S'_P contribution included.

TABLE VIII: Observables providing $\chi^2 \geq 1.5$ in at least one of the three fits with complex p'_P/p'_V .

Observable	$\gamma = 26^\circ$	$\gamma = 63^\circ$	$\gamma = 162^\circ$
$A_{CP}(B^+ \rightarrow \rho^+\pi^0)$	1.4	2.0	0.2
$\mathcal{B}(B^+ \rightarrow \rho^+\eta')$	2.4	3.5	2.8
$A_{CP}(B^0 \rightarrow \rho^-\pi^+)$	3.7	4.9	5.0
$A_{CP}(B^0 \rightarrow \rho^+\pi^-)$	0.4	1.0	1.7
$A_{CP}(B^+ \rightarrow \omega K^+)$	4.0	2.6	2.4
$S_{\phi K_S}$	1.6	1.6	1.6
Sum:	13.5	15.6	13.7

tion of the rates for $B \rightarrow K^*\eta^{(\prime)}$ is due in large part to the inclusion of the EWP contribution, interfering constructively in each case with the main QCD penguin term. Small CP asymmetries are predicted in $B^+ \rightarrow K^{*+}\eta$ and $B^0 \rightarrow K^{*0}\eta$. The considerable suppression of the decays involving η' reflects the destructive interference of the p'_P and p'_V contributions [31]. The predictions are presented in the absence of a singlet penguin (S'_V) contribution, which would primarily affect the $B \rightarrow K^*\eta'$ processes. For quantitative predictions as a function of $|S'_V/p'_P|$ see Ref. [12].

In the absence of electroweak penguin contributions one would expect the $B^+ \rightarrow \rho^0 K^+$ and $B^+ \rightarrow \omega K^+$ decay rates to be equal [70]. In fact, $\mathcal{B}(B^+ \rightarrow \omega K^+)/\mathcal{B}(B^+ \rightarrow \rho^0 K^+) = 1.3 \pm 0.3$, consistent with 1. This implies that additional electroweak penguin contributions in the form of the $2s'_P$ term in $A(B^+ \rightarrow \omega K^+)$

(see Table IV) do not considerably affect the branching ratio of this decay. A small but non-negligible positive CP asymmetry of about 0.2 is expected in both these processes through interference of the p'_V and $t'_V + c'_P$ terms.

The process $B^+ \rightarrow \rho^+ K^0$, as mentioned, is expected to be governed solely by the p'_V term. Measurement of its branching ratio would provide valued information on $|p'_V|$. As noted, the only upper limit on the branching ratio, $\mathcal{B}(B^+ \rightarrow \rho^+ K^0) < 48 \times 10^{-6}$, comes from the CLEO Collaboration, so it should be improved (or the decay discovered) very soon.

The rate difference sum rule (5) involving $B^0 \rightarrow K^{*+}\pi^-$ is seen in Eq. (7) to be satisfied, though with large errors. We predict small values for $A_{CP}(B^0 \rightarrow K^{*+}\pi^-)$ and $A_{CP}(B^0 \rightarrow \rho^+\pi^-)$.

We predict a branching ratio for $B^0 \rightarrow K^{*0}\pi^0$ of only about 10^{-6} , below the present experimental upper bound of 3.5×10^{-6} . The electroweak penguin component of the c'_V amplitude is responsible for interfering destructively with the p'_P component in this process. We expect a corresponding *enhancement* in $\mathcal{B}(B^+ \rightarrow K^{*+}\pi^0)$, since the relative signs of c'_V and the dominant penguin amplitude p'_P are opposite in the two processes. One expects the sum rule (analogous to one discussed recently in [7])

$$\mathcal{B}(B^+ \rightarrow K^{*+}\pi^0) + \frac{\tau_+}{\tau_0} \mathcal{B}(B^0 \rightarrow K^{*0}\pi^0) = \frac{1}{2} \left[\mathcal{B}(B^+ \rightarrow K^{*0}\pi^+) + \frac{\tau_+}{\tau_0} \mathcal{B}(B^0 \rightarrow K^{*+}\pi^-) \right] \quad (25)$$

to hold to first order in $|t'_P/p'_P|$ and $|c'_V/p'_P|$. The right-

hand side is $(12.7 \pm 2.2) \times 10^{-6}$. The sum rule is only approximately obeyed by the predicted branching ratios since quadratic terms in $|t'_P/p'_P|$ and $|c'_V/p'_P|$ are non-negligible.

The branching ratio and CP asymmetry for $B^0 \rightarrow \rho^- K^+$ are reproduced satisfactorily. Since these quantities enter into the rate difference relation (4), which is poorly obeyed [see (6)], one suspects that it is the experimental CP asymmetry $A_{CP}(B^0 \rightarrow \rho^- \pi^+) = -0.54 \pm 0.19$ which is slightly out of line, as mentioned earlier.

The decay $B^0 \rightarrow \rho^0 K^0$ is predicted to have a branching ratio of about 7×10^{-6} , not far below its experimental upper limit of 12×10^{-6} . The CP asymmetry is expected to be very small.

We already noted the comparison of $B^0 \rightarrow \omega K^0$ and $B \rightarrow \phi K$ amplitudes in Section V as one test for $p'_V = -p'_P$. Zero CP asymmetry is expected. We predict $A_{\phi K_S} = 0$, consistent with observation, but, as mentioned [65], are unable to account for $S_{\phi K_S} = -0.15 \pm 0.70$ ($S = 2.11$), predicting instead the value $\sin(2\beta) = 0.736 \pm 0.049$.

C. Processes sensitive to assumptions

Most parameters of the fits appear to be relatively stable. This stability is due in part to the inclusion of $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$, which are the only quantities in which the interference between t_V and t_P is probed directly. Small changes in relative strong phases occur when we relax the assumption that $p'_V/p'_P = -1$. The changes in predicted branching ratios and CP asymmetries appear to be so small that they will not be detected in the near future.

The least stable aspect of the fits is associated with the amplitudes C_V (color-suppressed tree) and $P'_{EW,V}$, contributing to s'_V and c'_V . The need for a C_V amplitude is associated with the large branching ratios for $B^+ \rightarrow \rho^+(\pi^0, \eta, \eta')$, but we still cannot fit the large branching ratio for the last process. The χ^2 minima at $\gamma \simeq (26, 162)^\circ$ are associated with larger values of C_V and $P'_{EW,V}$, which lead to the predictions $\mathcal{B}(B^+ \rightarrow K^{*+}\pi^0) = (22.1^{+4.2}_{-5.1}, 18.2^{+5.0}_{-4.1}) \times 10^{-6}$.

We have assumed no $SU(3)$ symmetry breaking in relating the $\Delta S = 0$ penguin amplitudes $|p_{P,V}|$ to the $|\Delta S| = 1$ amplitudes $|p'_{P,V}|$. This assumption will be checked in the future when the appropriate $B \rightarrow K^* \bar{K}$ or $B \rightarrow \bar{K}^* K$ decay rates are compared with the predictions in Table VI. The corresponding assumption $|P/P'| = |V_{cd}/V_{cs}| = 0.230$ is close to being checked in $B^+ \rightarrow K^+ \bar{K}^0$ decays, where it entails the prediction $\mathcal{B}(B^+ \rightarrow K^+ \bar{K}^0) = (0.75 \pm 0.11) \times 10^{-6}$ [13], to be compared with the experimental upper limit of 2.2×10^{-6} [71].

We have assumed nonet symmetry and a particular form of octet-singlet mixing in describing the decays $B \rightarrow K^* \eta$. When an independent measurement of the p'_V amplitude becomes available through the decay

$B^+ \rightarrow \rho^+ K^0$, this assumption will receive an independent check.

As mentioned earlier, we assumed that the strong phase of the electroweak penguin contribution $P'_{EW,P}$ to s'_P and c'_P is the same as that of p'_P , and that the $P'_{EW,V}$ contribution differs in phase by 180° with respect to p'_V . It may be necessary to relax these assumptions in future fits once more data become available involving these contributions.

VII. U-SPIN RELATIONS

In the previous sections we have employed the complete flavor $SU(3)$ symmetry group, neglecting small annihilation-type amplitudes. A best fit was performed in order to calculate magnitudes and phases of $SU(3)$ amplitudes. In the present section we will rely only on U-spin [72, 73], an important subgroup of $SU(3)$, introducing U-spin breaking in terms of ratios of decay constants. U-spin will be shown to imply two quadrangle relations among $|\Delta S| = 1$ amplitudes and two quadrangle relations among $\Delta S = 0$ amplitudes. This exhausts all sixteen $B^+ \rightarrow VP$ decays given in Tables III and IV. Relations will also be presented among penguin amplitudes in strangeness changing and strangeness conserving decays, and among tree amplitudes in these decays. Such relations may be used with branching ratio measurements to constrain tree amplitudes in $|\Delta S| = 1$ decays and penguin amplitudes in $\Delta S = 0$ decays. This could give an indication about potential CP asymmetries in certain modes. Expressions and values for decay amplitudes calculated in previous sections, where assumptions stronger than U-spin and U-spin breaking were made, must obey the quadrangle relations as well as these constraints.

The U-spin subgroup of $SU(3)$ is the same as the I-spin (isospin) except that the doublets with $U = 1/2, U_3 = \pm 1/2$ are

$$\text{Quarks : } \begin{bmatrix} |\frac{1}{2} & \frac{1}{2}\rangle \\ |\frac{1}{2} & -\frac{1}{2}\rangle \end{bmatrix} = \begin{bmatrix} |d\rangle \\ |s\rangle \end{bmatrix} , \quad (26)$$

$$\text{Antiquarks : } \begin{bmatrix} |\frac{1}{2} & \frac{1}{2}\rangle \\ |\frac{1}{2} & -\frac{1}{2}\rangle \end{bmatrix} = \begin{bmatrix} |\bar{s}\rangle \\ |-\bar{d}\rangle \end{bmatrix} . \quad (27)$$

B^+ is a U-spin singlet, while $\pi^+(\rho^+)$ and $K^+(K^{*+})$ belong to a U-spin doublet,

$$|0 \ 0\rangle = |B^+\rangle = |u\bar{b}\rangle , \quad (28)$$

$$\begin{bmatrix} |\frac{1}{2} & \frac{1}{2}\rangle \\ |\frac{1}{2} & -\frac{1}{2}\rangle \end{bmatrix} = \begin{bmatrix} |u\bar{s}\rangle = |K^+ (K^{*+})\rangle \\ |-\bar{u}d\rangle = -|\pi^+ (\rho^+)\rangle \end{bmatrix} . \quad (29)$$

Nonstrange neutral mesons belong either to a U-spin triplet or a U-spin singlet. The U-spin triplet residing in

the pseudoscalar meson octet is

$$\begin{bmatrix} |1 \ 1\rangle \\ |1 \ 0\rangle \\ |1 \ -1\rangle \end{bmatrix} = \begin{bmatrix} |K^0\rangle = |d\bar{s}\rangle \\ \frac{\sqrt{3}}{2}|\eta_8\rangle - \frac{1}{2}|\pi^0\rangle = \frac{1}{\sqrt{2}}|s\bar{s} - d\bar{d}\rangle \\ -|\bar{K}^0\rangle = -|s\bar{d}\rangle \end{bmatrix}, \quad (30)$$

and the corresponding singlet is

$$|0 \ 0\rangle = \frac{1}{2}|\eta_8\rangle + \frac{\sqrt{3}}{2}|\pi^0\rangle = \frac{1}{\sqrt{6}}|s\bar{s} + d\bar{d} - 2u\bar{u}\rangle. \quad (31)$$

In addition the η_1 is, of course, a U-spin singlet. We take $\eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$ and $\eta_1 \equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. The physical η and η' are mixtures of the octet and singlet,

$$\eta = \frac{2\sqrt{2}}{3}\eta_8 - \frac{1}{3}\eta_1, \quad \eta' = \frac{2\sqrt{2}}{3}\eta_1 + \frac{1}{3}\eta_8. \quad (32)$$

The U-spin triplet in the vector meson octet is

$$\begin{bmatrix} |1 \ 1\rangle \\ |1 \ 0\rangle \\ |1 \ -1\rangle \end{bmatrix} = \begin{bmatrix} |K^{*0}\rangle = |d\bar{s}\rangle \\ \frac{1}{\sqrt{2}}|\phi\rangle - \frac{1}{2}|\rho^0\rangle - \frac{1}{2}|\omega\rangle = \frac{1}{\sqrt{2}}|s\bar{s} - d\bar{d}\rangle \\ -|\bar{K}^{*0}\rangle = -|s\bar{d}\rangle \end{bmatrix}, \quad (33)$$

and the corresponding singlet is

$$|0 \ 0\rangle_8 = \frac{1}{\sqrt{6}}|\phi\rangle + \frac{\sqrt{3}}{2}|\rho^0\rangle - \frac{1}{2\sqrt{3}}|\omega\rangle = \frac{1}{\sqrt{6}}|s\bar{s} + d\bar{d} - 2u\bar{u}\rangle. \quad (34)$$

The SU(3) singlet vector meson is $|0 \ 0\rangle_1 = (|\phi\rangle + \sqrt{2}|\omega\rangle)/\sqrt{3}$.

The $\Delta C = 0$, $\Delta S = 1$ effective Hamiltonian transforms like a $\bar{s} \sim |\frac{1}{2} \frac{1}{2}\rangle$ component ($\Delta U_3 = \frac{1}{2}$) of a U-spin doublet, while the $\Delta C = 0$, $\Delta S = 0$ Hamiltonian transforms like a $\bar{d} \sim -|\frac{1}{2} - \frac{1}{2}\rangle$ component ($\Delta U_3 = -\frac{1}{2}$) of another U-spin doublet. Since the initial B^+ meson is a U-spin singlet, the final states are U-spin doublets. The VP states can be formed from a K^{*+} or a ρ^+ belonging to a U-spin doublet (29), while the pseudoscalar meson belongs to the two U-spin singlets, (31) and η_1 , and to the U-spin triplet (30). These VP states resemble the corresponding PP states studied within U-spin in Ref. [13], the only difference being that the K^+ and π^+ are now replaced by K^{*+} and ρ^+ . Alternatively, one may choose the pseudoscalar (K^+ or π^+) in a U-spin doublet, while the vector meson resides in the two U-spin singlets (34) and $|0 \ 0\rangle_1$ and in the triplet (33).

One finds four cases in each of which four physical amplitudes are expressed in terms of three U-spin amplitudes, corresponding to final states in which one of the final mesons is a member of a U-spin doublet while the other belongs to the two U-spin singlets and the U-spin triplet. This implies two quadrangle relations among $\Delta S = 1$ amplitudes and two quadrangle relations among $\Delta S = 0$ amplitudes:

$$\begin{aligned} 2\sqrt{2}A(K^{*+}\eta) + A(K^{*+}\eta') \\ = \sqrt{6}A(\rho^+K^0) + \sqrt{3}A(K^{*+}\pi^0), \end{aligned} \quad (35)$$

$$\begin{aligned} 2\sqrt{2}A(\rho^+\eta) + A(\rho^+\eta') \\ = \sqrt{3}A(\rho^+\pi^0) - \sqrt{6}A(K^{*+}\bar{K}^0), \end{aligned} \quad (36)$$

$$\begin{aligned} A(\rho^0K^+) + A(\omega K^+) \\ = \sqrt{2}A(\phi K^+) - \sqrt{2}A(K^{*0}\pi^+), \end{aligned} \quad (37)$$

$$\begin{aligned} A(\rho^0\pi^+) + A(\omega\pi^+) \\ = \sqrt{2}A(\phi\pi^+) + \sqrt{2}A(\bar{K}^{*0}K^+). \end{aligned} \quad (38)$$

The first two relations are straightforward generalizations of corresponding relations obtained for $B \rightarrow PP$ [13]. The last quadrangle is expected to be squashed, since the two terms on the right-hand-side contain no tree amplitude and are expected to be smaller than each of the two terms on the left-hand-side. (See expressions and values in Table III.) All four relations among complex amplitudes hold separately for B^+ and B^- decays.

One may decompose the $\Delta S = 1$ and $\Delta S = 0$ effective Hamiltonians into members of the *same* two U-spin doublets multiplying given CKM factors [73],

$$\mathcal{H}_{\text{eff}}^{\bar{b} \rightarrow \bar{s}} = V_{ub}^* V_{us} O_s^u + V_{cb}^* V_{cs} O_s^c, \quad (39)$$

$$\mathcal{H}_{\text{eff}}^{\bar{b} \rightarrow \bar{d}} = V_{ub}^* V_{ud} O_d^u + V_{cb}^* V_{cd} O_d^c. \quad (40)$$

Hadronic matrix elements of the two U-spin doublet operators, $O_{d,s}^u$ and $O_{d,s}^c$, will be denoted A^u and A^c and will be referred to as tree and penguin amplitudes, where the latter include electroweak penguin contributions. Note that these amplitudes multiply different CKM factors in $|\Delta S| = 1$ and $\Delta S = 0$ processes. The expressions (39) and (40) imply relations among penguin amplitudes A^c in strangeness changing and strangeness preserving processes and identical relations among corresponding tree amplitudes A^u .

Starting with processes involving η and η' , one may simply transcribe results obtained for $B \rightarrow PP$ [13], replacing π^+ and K^+ by ρ^+ and K^{*+} . Thus, one finds expressions for $\Delta S = 0$ penguin amplitudes in terms of sums of two $|\Delta S| = 1$ penguin amplitudes which are expected to dominate these processes [13],

$$A^c(\rho^+\eta) = A^c(K^{*+}\eta) - \frac{2}{\sqrt{3}}A^c(\rho^+K^0), \quad (41)$$

$$A^c(\rho^+\eta') = A^c(K^{*+}\eta') - \frac{1}{\sqrt{6}}A^c(\rho^+K^0). \quad (42)$$

Since all amplitudes involve unknown strong phases, these are in general triangle relations. Assuming that the two penguin amplitudes on the right-hand sides of each of Eqs. (41) and (42) dominate the respective processes, the rates of these processes may be used to obtain constraints on the penguin amplitudes on the left hand sides. For this purpose one would need to measure $\mathcal{B}(B^+ \rightarrow \rho^+K^0)$ and improve the upper bound on $\mathcal{B}(B^+ \rightarrow K^{*+}\eta')$.

Similarly, one obtains expressions for $\Delta S = 1$ tree amplitudes in terms of sums of two $\Delta S = 0$ tree amplitudes,

$$A^u(K^{*+}\eta) = A^u(\rho^+\eta) + \frac{2}{\sqrt{3}}A^u(K^{*+}\bar{K}^0), \quad (43)$$

$$A^u(K^{*+}\eta') = A^u(\rho^+\eta') + \frac{1}{\sqrt{6}}A^u(K^{*+}\bar{K}^0) . \quad (44)$$

The second terms on the right-hand-sides vanish in the approximation of neglecting annihilation amplitudes. This provides two equalities between tree amplitudes in $B^+ \rightarrow K^{*+}\eta$ (η') and $B^+ \rightarrow \rho^+\eta$ (η'). (In Tables III and IV these amplitudes are given by t'_P and t_P , respectively.) The observed amplitude $A(B^+ \rightarrow \rho^+\eta) = 31.2 \pm 4.7$ eV (see Table III) then implies, via Eq. (43), that the tree contribution in $B^+ \rightarrow K^{*+}\eta$ is $(31.2 \pm 4.7) \times \bar{\lambda}(f_{K^*}/f_\rho) \simeq (8 \pm 1)$ eV. This is approximately the value calculated in Table IV.

Another set of U-spin relations, applying separately to penguin and tree amplitudes, can be derived for decay amplitudes involving ρ^0 , ω and ϕ . Physical amplitudes, consisting of penguin and tree contributions, may be decomposed into U-spin amplitudes,

$$3A(\omega\pi^+) = -A_0^d - 3A_1^d + \sqrt{2}B_0^d , \quad (45)$$

$$3A(\phi\pi^+) = \sqrt{2}A_0^d + 3\sqrt{2}A_1^d + B_0^d , \quad (46)$$

$$3A(\omega K^+) = -A_0^s + 3A_1^s + \sqrt{2}B_0^s , \quad (47)$$

$$3A(\phi K^+) = \sqrt{2}A_0^s - 3\sqrt{2}A_1^s + B_0^s , \quad (48)$$

$$A(\rho^0\pi^+) = A_0^d - A_1^d , \quad (49)$$

$$A(\bar{K}^{*0}K^+) = -2\sqrt{2}A_1^d , \quad (50)$$

$$A(\rho^0K^+) = A_0^s + A_1^s , \quad (51)$$

$$A(K^{*0}\pi^+) = -2\sqrt{2}A_1^s , \quad (52)$$

where A_0 , A_1 and B_0 correspond to final states with vector mesons in U-spin singlet and triplet in the octet and in the SU(3) singlet, respectively. The superscripts d and s denote strangeness conserving and strangeness changing amplitudes, respectively.

This decomposition implies several relations for penguin amplitudes,

$$A^c(\rho^0\pi^+) = A^c(\rho^0K^+) + A^c(K^{*0}\pi^+)/\sqrt{2} , \quad (53)$$

$$A^c(\omega\pi^+) = A^c(\omega K^+) + A^c(K^{*0}\pi^+)/\sqrt{2} , \quad (54)$$

$$A^c(\phi\pi^+) = A^c(\phi K^+) - A^c(K^{*0}\pi^+) , \quad (55)$$

$$A^c(\bar{K}^{*0}K^+) = A^c(K^{*0}\pi^+) . \quad (56)$$

The penguin amplitudes on the right-hand-sides dominate the corresponding processes. Assuming $p'_P = -p'_V$, the two terms on the right-hand-side of Eq. (53), $-p'_V/\sqrt{2} + p'_P/\sqrt{2}$ add up constructively, while the two terms in Eq. (54), $(p'_V + 2s'_P)/\sqrt{2} + p'_P/\sqrt{2}$, add up destructively. Destructive interference occurs also in Eq. (55), where the right-hand-side is $(p'_P + s'_P) - p'_P$. Consequently, one expects large tree-penguin interference in $B^+ \rightarrow \rho^0\pi^+$, and small interference in decays to $\omega\pi^+$ and $\phi\pi^+$. No CP asymmetries are expected in the two processes of Eq. (56) which are pure penguin in our approximation.

Similarly, one obtains U-spin relations for tree amplitudes,

$$A^u(\rho^0K^+) = A^u(\rho^0\pi^+) - A^u(\bar{K}^{*0}K^+)/\sqrt{2} , \quad (57)$$

$$A^u(\omega K^+) = A^u(\omega\pi^+) - A^u(\bar{K}^{*0}K^+)/\sqrt{2} , \quad (58)$$

$$A^u(\phi K^+) = A^u(\phi\pi^+) + A^u(\bar{K}^{*0}K^+) , \quad (59)$$

$$A^u(K^{*0}\pi^+) = A^u(\bar{K}^{*0}K^+) . \quad (60)$$

In the approximation of neglecting annihilation amplitudes the second terms in Eqs. (57)–(59) vanish. Thus, tree amplitudes within each of the three pairs of $\Delta S = 0$ and $|\Delta S| = 1$ processes involving a K^+ and a π^+ are equal. Assuming that the tree amplitude dominates $B^+ \rightarrow \omega\pi^+$, where the penguin amplitudes p_P and p_V interfere destructively, Eq. (58) implies a sizable tree amplitude in $B^+ \rightarrow \omega K^+$, $\approx 23 \text{ eV} \times \bar{\lambda}(f_K/f_\pi) \simeq 7 \text{ eV}$. This value, calculated earlier in Table IV, permits a sizable tree-penguin interference in this decay. Table IV shows equal tree amplitudes in $B^+ \rightarrow \omega K^+$ and $B^+ \rightarrow \rho^0 K^+$. This result is beyond U-spin. All the tree amplitudes in Eqs. (59) and (60) vanish in our approximation.

VIII. CONCLUSIONS

We have analyzed the decays of B mesons to a charmless vector (V) and pseudoscalar (P) meson in the framework of flavor SU(3). The relative magnitudes of tree and color-suppressed amplitudes extracted from data appear consistent with the factorization hypothesis. For example, the ratio of the tree amplitude in which the current produces a vector meson to that in which it produces a pseudoscalar is approximately f_ρ/f_π , and the ratio of color-suppressed to tree amplitudes is approximately that in $B \rightarrow PP$ data.

Penguin amplitudes are also extracted from data. Here we are not aware of successful *a priori* predictions of their magnitudes. For solutions compatible with other determinations of γ [69], we find a fairly stable pattern of small final-state phases (mod π), implying small CP asymmetries in all processes. In particular, we do not expect a large CP asymmetry in $B^0 \rightarrow \rho^-\pi^+$. We find a small relative strong phase between t_V and t_P . There exist also solutions for γ outside the expected range; these have larger final-state phases but cannot be excluded by present experiments. Our preferred $\gamma \simeq 63^\circ$ fit favors a weak phase γ within the range 57° – 69° at the 1σ level, and 51° – 73° at 95% c.l. if one restricts attention to the range 38° – 80° allowed in fits to other data [69].

Predictions have been made for rates and CP asymmetries in as-yet-unseen decay modes. Some of these modes, such as $B^+ \rightarrow \rho^+K^0$, $B^0 \rightarrow \rho^0K^0$, and $B^+ \rightarrow K^{*+}\pi^0$ should be seen soon.

A key assumption for which we have performed some tests and suggested others is the relation [31] $p'_V/p'_P \simeq -1$ between penguin amplitudes in which the spectator quark is incorporated into either a pseudoscalar meson or a vector meson. This relation is quite well satisfied, with the question of a small relative strong phase between p'_V and $-p'_P$ still open.

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APPENDIX: $B \rightarrow \rho^\mp \pi^\pm$ RATES AND ASYMMETRIES

In Table I the CP -averaged branching ratio $\mathcal{B}_{\rho\pi}^{\pm\mp}$ quoted for the decay $B^0 \rightarrow \rho^\mp \pi^\pm$ is the sum of the CP -averaged branching ratios for $B^0 \rightarrow \rho^-\pi^+$ and $B^0 \rightarrow \rho^+\pi^-$. The CP asymmetry $A_{CP}(\rho^\mp \pi^\pm) = -0.14 \pm 0.08 \equiv A_{\rho\pi}$ is

$$A_{\rho\pi} = \frac{\mathcal{B}(\rho^+\pi^-) - \mathcal{B}(\rho^-\pi^+)}{\mathcal{B}(\rho^+\pi^-) + \mathcal{B}(\rho^-\pi^+)} , \quad (61)$$

where

$$\mathcal{B}(\rho^\pm \pi^\mp) \equiv \mathcal{B}(B^0 \rightarrow \rho^\pm \pi^\mp) + \mathcal{B}(\bar{B}^0 \rightarrow \rho^\pm \pi^\mp) . \quad (62)$$

These quantities are related to the individual CP -averaged branching ratios and CP asymmetries by [29, 50]

$$\begin{aligned} & \frac{1}{2}[\mathcal{B}(B^0 \rightarrow \rho^\pm \pi^\mp) + \mathcal{B}(\bar{B}^0 \rightarrow \rho^\mp \pi^\pm)] \\ &= \frac{1}{2}(1 \pm \Delta C \pm A_{\rho\pi} C) \mathcal{B}_{\rho\pi}^{\pm\mp} , \end{aligned} \quad (63)$$

where $C = 0.35 \pm 0.13 \pm 0.05$ and $\Delta C = 0.20 \pm 0.13 \pm 0.05$ are measured in time-dependent decays [48, 50]. The individual CP asymmetries are

$$A_{CP}(B^0 \rightarrow \rho^-\pi^+) = \frac{A_{\rho\pi} - C - A_{\rho\pi} \Delta C}{1 - \Delta C - A_{\rho\pi} C} , \quad (64)$$

$$A_{CP}(B^0 \rightarrow \rho^+\pi^-) = -\frac{A_{\rho\pi} + C + A_{\rho\pi} \Delta C}{1 + \Delta C + A_{\rho\pi} C} . \quad (65)$$

In calculating the entries in Table I for the individual branching ratios and asymmetries we have used the correlations among the input variables [48] to evaluate the experimental errors.

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